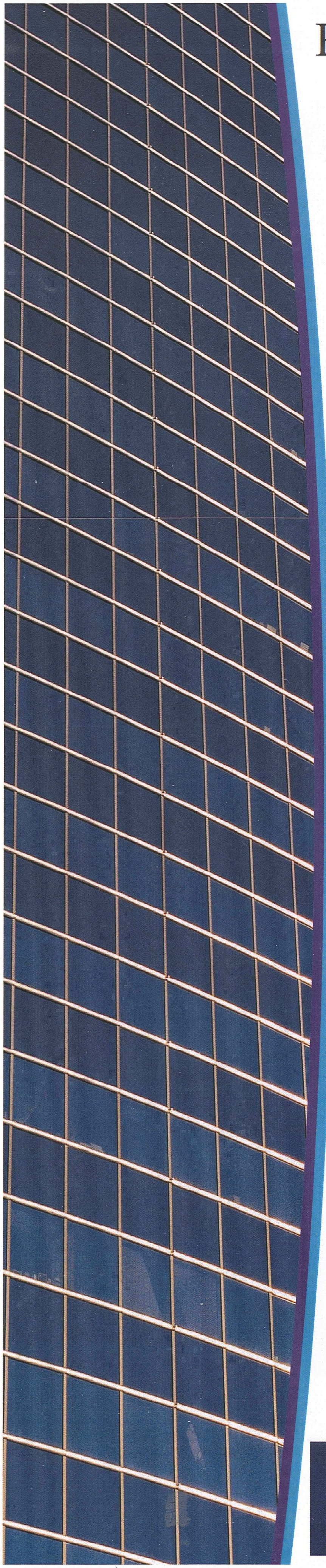

Providence, RI

June 3-7, 2013

# ILAS 2013

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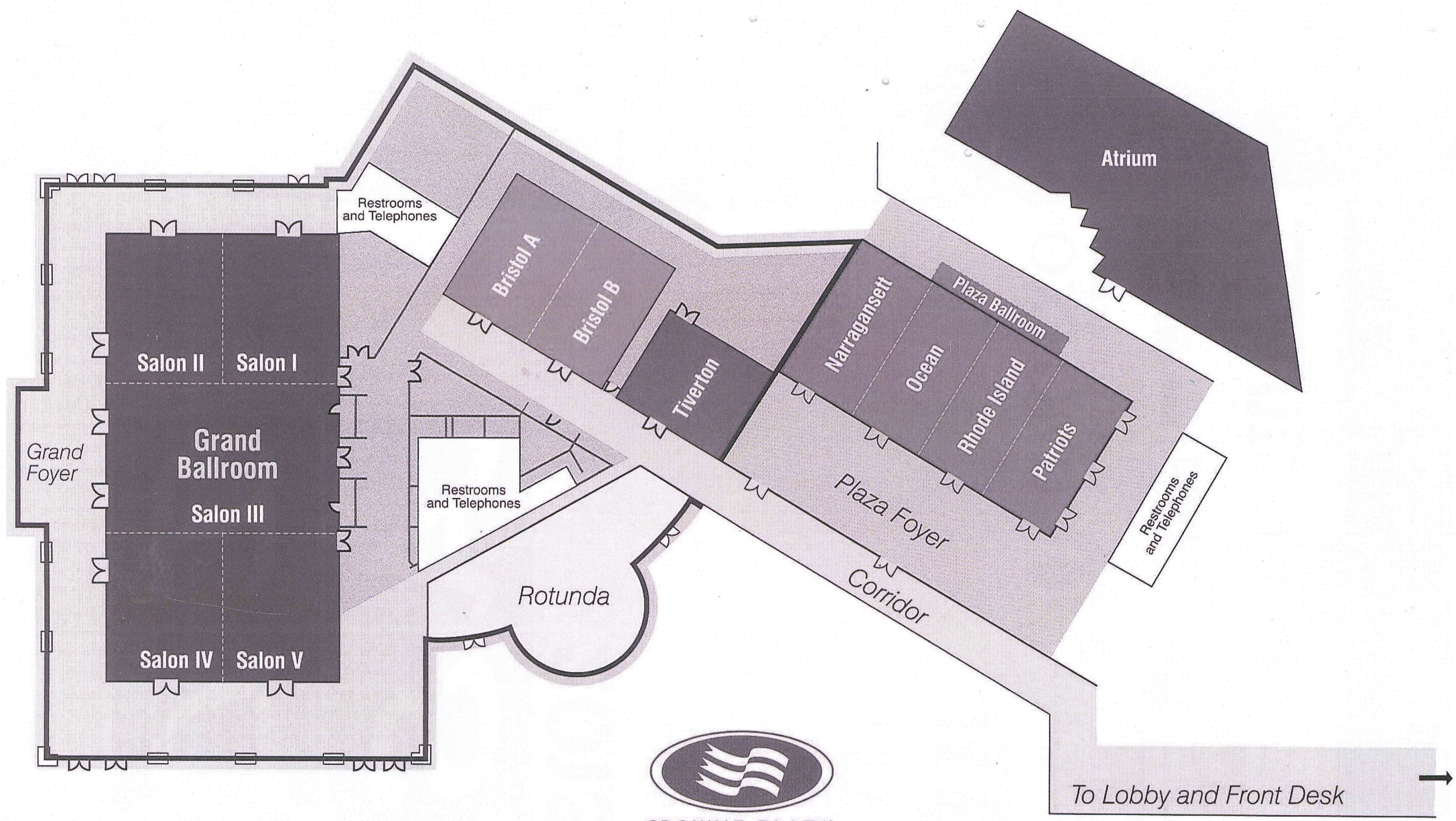


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# ILAS 2013

Providence, RI, June 3-7, 2013

The 18th Conference of the International Linear Algebra Society

## Local Organizing Committee

- Tom Bella (chair), University of Rhode Island
- Misha Kilmer, Tufts University
- Steven Leon, UMass Dartmouth
- Vadim Olshevsky, University of Connecticut

## Scientific Organizing Committee

- Tom Bella (chair)
- Vadim Olshevsky (chair)
- Ljiljana Cvetkovic
- Heike Fassbender
- Chen Greif
- J. William Helton
- Olga Holtz
- Steve Kirkland
- Victor Pan
- Panayiotis Psarrakos
- Tin Yau Tam
- Paul Van Dooren
- Hugo Woerdeman

## Conference Staff

- Kate Bella
- Bill Jamieson
- Michael Krul
- Heath Loder
- Caitlin Phifer
- Jenna Reis
- Daniel Richmond
- Diana Smith
- Daniel Sorenson

# Contents

# Conference Information

## Host Institution and Conference Venue

The University of Rhode Island, the state's current Land, Sea and Urban Grant public research institution, is the host institution for the conference. The conference venue is the Crowne Plaza Providence–Warwick. All talks, events, and the meeting banquet will be held at the Crowne Plaza. Located just minutes from downtown Providence, the Crowne Plaza is also easily accessed from the airport, as they offer a free, 24 hour shuttle service to and from Green Airport. The address is

Crowne Plaza Providence–Warwick
801 Greenwich Ave
Warwick, Rhode Island 02886

## Travel Information

**TF Greene International Airport** (Airport Code PVD) - Green is located two miles away from the Crowne Plaza, and less than a mile from the Holiday Inn Express (both hotels offer a free 24 hour shuttle to and from TF Green). Details on ground transportation from Green may be found at `http://www.pvdairport.com/main.aspx?sec_id=59`.

**Logan International Airport** (Boston) (Airport Code BOS) - Logan Airport is 62 miles from the Crowne Plaza, and is easily reachable by car via I-95 South, or by train into Providence (see below).

**Trains (MBTA Commuter Rail, Amtrak, etc.)** - The Providence Train Station is located downtown, 12 miles from the Crowne Plaza. The hotel may be reached by taxi.

## Registration

The registration desk is located outside of the Grand Ballroom, near the Rotunda (see map). The hours of operation of the registration desk are:

| | | | |
|---|---|---|---|
| Sunday, June 2nd | 6:00pm | - | 8:00pm |
| Monday, June 3rd | 7:00am | - | 12:00pm |
| | 1:30pm | - | 7:00pm |
| Tuesday, June 4th | 7:00am | - | 12:00pm |
| | 1:30pm | - | 3:30pm |
| Wednesday, June 5th | 8:00am | - | 10:00am |
| Thursday, June 6th | 8:00am | - | 10:00am |
| | 1:30pm | - | 3:30pm |
| Friday, June 6th | 8:00am | - | 10:00am |

Attendees who have already paid the registration fee may pick up their materials by dropping by the registration desk. If you have not yet paid the registration fee, you may do so at the registration desk by cash or check (in US dollars).

If you have any questions, please stop by and ask our staff.

## WiFi Internet Connection

Wireless internet is available throughout the meeting spaces in the Crowne Plaza to all attendees. The wireless access point to connect to is **crowne1** (the last character is the numeral "one") and the password (which is *case–sensitive*) is **Advantage7**. Guests at the Crowne Plaza should note that this access information may be different than the information in guestrooms.

## Presentation Equipment

All meeting rooms are equipped with a high–lumen projector and large projection screen, together with a laptop computer and remote control/laser pointer. The provided laptop may be used to present a talk on a USB jumpdrive in either PDF or PPT format (though in general not all PPT formats seem to be compatible with all PPT viewers). Presenters may also connect their own computers directly to the projector.

We encourage all presenters to preload the PDF for their talk onto the laptop computer beforehand, and ensure that it is displaying correctly. Presenters using their own laptops should check compatibility with the projector in advance, if possible.

## Coffee Breaks, Lunches, and Dinners

Coffee, tea, and juice will be available in the Grand Foyer each morning from 7:15am until 8:15am. After the morning plenary sessions, there will be a coffee break with snacks from 9:30am until 10am. The afternoon coffee breaks will be held between the afternoon parallel sessions, and concurrent with the ILAS Business Meeting on Thursday.

During lunch and dinner, attendees will be on their own. Some options are:

- Concession Lunches – Grand Foyer – The Crowne Plaza will have lunches available for sale in the Grand Foyer for ILAS attendees.
- Remington's Restaurant – Guests can enjoy fine American Cuisine and a friendly dining atmosphere without ever leaving the hotel. Remington's Restaurant serves breakfast, lunch and dinner.
- Alfred's Lounge – Alfred's Lounge offers a traditional library style decor with oversized luscious leather sofas and a well appointed fireplace. Alfred's Lounge features over 100 wine selections by the glass. Also serving dinner and light fare.
- Area Restaurants – The Crowne Plaza operates a shuttle service for small groups to nearby restaurants. Contact the hotel front desk (not the ILAS registration desk) for details.
- Room Service is also available 24 hours.

## Welcome Reception

There will be a welcome reception for attendees arriving the night before the conference, Sunday June 2nd, from 6:00pm until 8:00pm. The welcome reception will be in the Rotunda. If you are arriving on Sunday or earlier, please stop by! The registration desk will be open during this time as well, so you may check in and pick up your attendee materials early if you wish.

## Conference Excursion

The Newport Mansions are a major tourist attraction in Rhode Island. Details may be found on their website at `http://www.newportmansions.org/`. The excursion to Newport will be Wednesday afternoon, and the $25 fee includes bus transportation to and from Newport and admission into the mansion visited. Due to high demand, tickets for the excursion are almost sold out at this time, but any remaining tickets may be purchased online or at the registration desk (with cash or checks in US Dollars) while supplies last.

Attendees with tickets will have the opportunity to sign up to visit either **The Breakers** or **Rosecliff**. Two buses will be going to The Breakers, and one bus will be going to Rosecliff. If you have a preference of which mansion you'd like to visit, please let us know at the registration desk.

**The Breakers**

The Breakers is the grandest of Newport's summer "cottages" and a symbol of the Vanderbilt family's social and financial preeminence in turn of the century America. Commodore Cornelius Vanderbilt (1794-1877) established the family fortune in steamships and later in the New York Central Railroad, which was a pivotal development in the industrial growth of the nation during the late 19th century.

**Rosecliff**

Commissioned by Nevada silver heiress Theresa Fair Oelrichs in 1899, architect Stanford White modeled Rosecliff after the Grand Trianon, the garden retreat of French kings at Versailles. Scenes from several films have been shot on location at Rosecliff, including *The Great Gatsby*, *True Lies*, *Amistad* and *27 Dresses*.

(descriptions and photographs courtesy of The Preservation Society of Newport County)

**Tentative Schedule for the Newport Excursion**

| | | |
|---|---|---|
| Wednesday | 1:00pm | Meet buses outside of Crowne Plaza |
| Wednesday | 2:45pm | Arrive at The Breakers or Rosecliff<br>Explore Mansion and grounds, 90 minutes |
| Wednesday | 4:30pm | Meet back at buses |
| Wednesday | 5:00pm | Arrive in downtown Newport<br>Attendees on their own for dinner |
| Wednesday | 8:00pm | Meet back at buses |
| Wednesday | 9:30pm | Arrive back at Crowne Plaza |

After visiting the mansions, attendees will have the opportunity to visit the downtown Newport area for dinner (attendees on their own). A (very incomplete) list of some local restaurants is given below.

**Some Restaurants in Downtown Newport**

| | | |
|---|---|---|
| Black Pearl | 401.846.5264 | `blackpearlnewport.com` |
| Brick Alley Pub and Restaurant | 401.849.6334 | `brickalley.com` |
| Busker's Pub and Restaurant | 401.846.5856 | `buskerspub.com` |
| Fluke Wine Bar and Kitchen | 401.849.7778 | `flukewinebar.com` |
| Gas Lamp Grille | 401.845.9300 | `gaslampgrille.com` |
| Lucia Italian Restaurant and Pizzeria | 401.846.4477 | `luciarestaurant.com` |
| Panera Bread | 401.324.6800 | `panerabread.com` |
| The Barking Crab | 401.846.2722 | `barkingcrab.com` |
| Wharf Pub and Restaurant | 401.846.9233 | `thewharfpub.com` |

## Conference Banquet

The conference banquet will be held on Thursday night, June 6th, from 8pm until 10:30pm. Tickets for the banquet are not included in the conference registration, and may be purchased in advance online through the registration system for $50 each, or at the registration desk (with cash or checks in US Dollars) while supplies last. During the banquet, the Hans Schneider Prize will be presented to Thomas Laffey.

## ILAS Membership Business Meeting

ILAS will hold its Membership Business Meeting on Thursday afternoon, 3:30pm – 5:30pm, in the Grand Ballroom.

## Conference Proceedings

The proceedings of the 18th Conference of the International Linear Algebra Society, Providence, USA, June 3-7, 2013, will appear as a special issue of Linear Algebra and Its Applications.

The Guest Editors for this issue are:

- Chen Greif,
- Panayiotis Psarrakos,
- Vadim Olshevsky, and
- Hugo J. Woerdeman.

The responsible Editor-in-Chief of the special issue is Peter Semrl.

The deadline for submissions will be October 31, 2013.

All papers submitted for the conference proceedings will be subject to the usual LAA refereeing process.

They should be submitted via the Elsevier Editorial System (EES): http://ees.elsevier.com/laa/

When selecting Article Type please choose Special Issue: 18th ILAS Conference and when requesting an editor please select the responsible Editor-in-Chief P. Semrl. You will have a possibility to suggest one of the four guest editors that you would like to handle your paper.

ILAS 2013 Schedule Overview

| | | | | |
|---|---|---|---|---|
| Opening Remarks | | | | |
| Anne Greenbaum | Raymond Sze | Thomas Laffey | Fuzhen Zhang | Dan Spielman |
| Leiba Rodman | Maryam Fazel | Gilbert Strang | Dianne O'Leary | Jean Bernard Lasserre |
| Coffee Break | Coffee Break | Coffee Break | Coffee Break | Coffee Break |
| Linear Algebra Problems in Quantum Computation<br>Nonlinear Eigenvalue Problems<br>Advances in Combinatorial Matrix Theory and its Ap...<br>Generalized Inverses and Applications<br>Linear Least Squares Methods: Algorithms, Analysis... | Linear Algebra Problems in Quantum Computation<br>Randomized Matrix Algorithms<br>Matrix Methods for Polynomial Root-Finding<br>Multilinear Algebra and Tensor Decompositions<br>Linear Algebra, Control, and Optimization<br>Contributed Session on Computational Science | Matrices and Graph Theory<br>Symbolic Matrix Algorithms<br>Matrices and Orthogonal Polynomials<br>Structure and Randomization in Matrix Computations<br>Contributed Session on Numerical Range and Spectra... | Matrices and Graph Theory<br>Symbolic Matrix Algorithms<br>Matrices and Orthogonal Polynomials<br>Structure and Randomization in Matrix Computations<br>Contributed Session on Matrix Pencils<br>Contributed Session on Numerical Linear Algebra | Matrices and Orthogonal Polynomials<br>Abstract Interpolation and Linear Algebra<br>Linear Algebra Education Issues<br>Matrix Inequalities<br>Sign Pattern Matrices<br>Contributed Session on Nonnegative Matrices |
| Lunch | Lunch | Lunch | Lunch | Lunch |
| Alan Edelman | Ivan Oseledets | Excursion: Newport Mansions | | Ravi Kannan |
| Linear Algebra Problems in Quantum Computation<br>Krylov Subspace Methods for Linear Systems<br>Nonlinear Eigenvalue Problems<br>Advances in Combinatorial Matrix Theory and its Ap...<br>Generalized Inverses and Applications<br>Contributed Session on Algebra and Matrices over A... | Matrices and Graph Theory<br>Matrix Methods for Polynomial Root-Finding<br>Multilinear Algebra and Tensor Decompositions<br>Linear Complementarity Problems and Beyond<br>Linear Algebra, Control, and Optimization<br>Contributed Session on Numerical Linear Algebra | | Matrices and Orthogonal Polynomials<br>Linear Algebra Education Issues<br>Structure and Randomization in Matrix Computations<br>Linear and Nonlinear Perron-Frobenius Theory<br>Contributed Session on Graph Theory<br>Contributed Session on Nonnegative Matrices | Structured Matrix Functions and their Applications...<br>Linear Algebra Education Issues<br>Matrix Inequalities<br>Matrices and Total Positivity<br>Contributed Session on Positive Definite Matrices |
| Coffee Break | Coffee Break | | ILAS Business Meeting | Coffee Break |
| Krylov Subspace Methods for Linear Systems<br>Nonlinear Eigenvalue Problems<br>Matrices over Idempotent Semirings<br>Advances in Combinatorial Matrix Theory and its Ap...<br>Contributed Session on Algebra and Matrices over A...<br>Linear Algebra Problems in Quantum Computation | Matrices and Graph Theory<br>Matrix Methods for Polynomial Root-Finding<br>Multilinear Algebra and Tensor Decompositions<br>Applications of Tropical Mathematics<br>Contributed Session on Numerical Linear Algebra | | Abstract Interpolation and Linear Algebra<br>Structured Matrix Functions and their Applications...<br>Matrix Inequalities<br>Linear and Nonlinear Perron-Frobenius Theory<br>Sign Pattern Matrices<br>Contributed Session on Matrix Completion Problems | Linear Algebra Problems in Quantum Computation<br>Structured Matrix Functions and their Applications...<br>Applications of Tropical Mathematics<br>Matrix Methods in Computational Systems Biology an...<br>Matrices and Total Positivity<br>Contributed Session on Matrix Equalities and Inequ... |
| | ILAS Board Meeting | | Banquet | Closing Remarks |

# Sunday

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|---|---|
| **Sunday, 06:00PM - 08:00PM**<br>**Location: Rotunda** | **Welcome Reception and Registration** |

# Monday

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|---|---|
| **Monday, 08:00AM - 08:45AM**<br>**Location: Grand Ballroom** | **Anne Greenbaum (University of Washington) (SIAG LA speaker*)**<br>Session Chair: Stephen Kirkland |
| **Monday, 08:45AM - 09:30AM**<br>**Location: Grand Ballroom** | **Leiba Rodman (College of William and Mary) (LAMA Lecture)**<br>Session Chair: Stephen Kirkland |

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|---|---|
| **Monday, 09:30AM - 10:00AM**<br>**Location: Grand Foyer** | **Coffee Break** |

| | |
|---|---|
| **Monday, 10:00AM - 12:00PM**<br>**Location: Grand Ballroom** | **Linear Algebra Problems in Quantum Computation**<br>10:00 - 10:30 – Jinchuan Hou<br>10:30 - 11:00 – Lajos Molnar<br>11:00 - 11:30 – Zejun Huang<br>11:30 - 12:00 – Gergo Nagy |
| **Monday, 10:00AM - 12:00PM**<br>**Location: Bristol A** | **Advances in Combinatorial Matrix Theory and its Applications**<br>10:00 - 10:30 – Carlos da Fonseca<br>10:30 - 11:00 – Ferenc Szollosi<br>11:00 - 11:30 – Geir Dahl<br>11:30 - 12:00 – Judi McDonald |
| **Monday, 10:00AM - 12:00PM**<br>**Location: Bristol B** | **Generalized Inverses and Applications**<br>10:00 - 10:30 – Minnie Catral<br>10:30 - 11:00 – Nieves Castro-Gonzalez<br>11:00 - 11:30 – Esther Dopazo |
| **Monday, 10:00AM - 12:00PM**<br>**Location: Ocean** | **Nonlinear Eigenvalue Problems**<br>10:00 - 10:30 – Francoise Tisseur<br>10:30 - 11:00 – Karl Meerbergen<br>11:00 - 11:30 – Elias Jarlebring<br>11:30 - 12:00 – Zhaojun Bai |
| **Monday, 10:00AM - 12:00PM**<br>**Location: Patriots** | **Linear Least Squares Methods: Algorithms, Analysis, and Applications**<br>10:00 - 10:30 – David Titley-Peloquin<br>10:30 - 11:00 – Huaian Diao<br>11:00 - 11:30 – Sanzheng Qiao<br>11:30 - 12:00 – Keiichi Morikuni |

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|---|---|
| **Monday, 12:00PM - 01:20PM**<br>**Location: Grand Foyer** | **Lunch - Attendees on their own - Lunch on sale in Grand Foyer** |

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|---|---|
| **Monday, 01:20PM - 02:05PM**<br>**Location: Grand Ballroom** | **Alan Edelman (MIT)**<br>Session Chair: Ilse Ipsen |

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|---|---|
| **Monday, 02:15PM - 04:15PM**<br>**Location: Grand Ballroom** | **Linear Algebra Problems in Quantum Computation**<br>2:15 - 2:45 – David Kribs<br>2:45 - 3:15 – Jianxin Chen<br>3:15 - 3:45 – Nathaniel Johnston<br>3:45 - 4:15 – Sarah Plosker |
| **Monday, 02:15PM - 04:15PM**<br>**Location: Bristol A** | **Advances in Combinatorial Matrix Theory and its Applications**<br>2:15 - 2:45 – Gi-Sang Cheon<br>2:45 - 3:15 – Kathleen Kiernan<br>3:15 - 3:45 – Kevin Vander Meulen<br>3:45 - 4:15 – Ryan Tifenbach |
| **Monday, 02:15PM - 04:15PM**<br>**Location: Bristol B** | **Generalized Inverses and Applications**<br>2:15 - 2:45 – Huaian Diao<br>2:45 - 3:15 – Chunyuan Deng<br>3:15 - 3:45 – Dragana Cvetkovic-Ilic<br>3:45 - 4:15 – Dijana Mosic |
| **Monday, 02:15PM - 04:15PM**<br>**Location: Tiverton** | **Contributed Session on Algebra and Matrices over Arbitrary Fields Part 1**<br>2:15 - 2:35 – Polona Oblak<br>2:35 - 2:55 – Gregor Dolinar<br>3:15 - 3:35 – Leo Livshits<br>3:35 - 3:55 – Peter Semrl |
| **Monday, 02:15PM - 04:15PM**<br>**Location: Ocean** | **Nonlinear Eigenvalue Problems**<br>2:15 - 2:45 – Ion Zaballa<br>2:45 - 3:15 – Shreemayee Bora<br>3:15 - 3:45 – Leo Taslaman<br>3:45 - 4:15 – Christian Mehl |
| **Monday, 02:15PM - 04:15PM**<br>**Location: Patriots** | **Krylov Subspace Methods for Linear Systems**<br>2:15 - 2:45 – Eric de Sturler<br>2:45 - 3:15 – Daniel Richmond<br>3:15 - 3:45 – Andreas Stathopoulos<br>3:45 - 4:15 – Daniel Szyld |

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|---|---|
| **Monday, 04:15PM - 04:45PM**<br>**Location: Grand Foyer** | **Coffee Break** |

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|---|---|
| **Monday, 04:45PM - 06:45PM**<br>**Location: Grand Ballroom** | **Linear Algebra Problems in Quantum Computation**<br>4:45 - 5:15 – Tin-Yau Tam<br>5:15 - 5:45 – Diane Christine Pelejo<br>5:45 - 6:15 – Justyna Zwolak |
| **Monday, 04:45PM - 06:45PM**<br>**Location: Bristol A** | **Advances in Combinatorial Matrix Theory and its Applications**<br>4:45 - 5:15 – In-Jae Kim<br>5:15 - 5:45 – Charles Johnson |
| **Monday, 04:45PM - 06:45PM**<br>**Location: Bristol B** | **Matrices over Idempotent Semirings**<br>4:45 - 5:15 – Sergei Sergeev<br>5:15 - 5:45 – Jan Plavka<br>5:45 - 6:15 – Hans Schneider<br>6:15 - 6:45 – Martin Gavalec |
| **Monday, 04:45PM - 06:45PM**<br>**Location: Tiverton** | **Contributed Session on Algebra and Matrices over Arbitrary Fields Part 2**<br>4:45 - 5:05 – Mikhail Stroev<br>5:05 - 5:25 – Bart De Bruyn<br>5:25 - 5:45 – Rachel Quinlan<br>5:45 - 6:05 – Yorick Hardy |
| **Monday, 04:45PM - 06:45PM**<br>**Location: Ocean** | **Nonlinear Eigenvalue Problems**<br>4:45 - 5:15 – D. Steven Mackey<br>5:15 - 5:45 – Fernando De Teran<br>5:45 - 6:15 – Vanni Noferini<br>6:15 - 6:45 – Federico Poloni |
| **Monday, 04:45PM - 06:45PM**<br>**Location: Patriots** | **Krylov Subspace Methods for Linear Systems**<br>4:45 - 5:15 – James Baglama<br>5:15 - 5:45 – Jennifer Pestana<br>5:45 - 6:15 – Xiaoye Li |

# Tuesday

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|---|---|
| **Tuesday, 08:00AM - 08:45AM**<br>**Location: Grand Ballroom** | **Raymond Sze (Hong Kong Polytechnic University)**<br>Session Chair: Tin-Yau Tam |
| **Tuesday, 08:45AM - 09:30AM**<br>**Location: Grand Ballroom** | **Maryam Fazel (University of Washington)**<br>Session Chair: Tin-Yau Tam |

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|---|---|
| **Tuesday, 09:30AM - 10:00AM**<br>**Location: Grand Foyer** | **Coffee Break** |

| | |
|---|---|
| **Tuesday, 10:00AM - 12:00PM**<br>**Location: Grand Ballroom** | **Linear Algebra Problems in Quantum Computation**<br>10:00 - 10:30 – Edward Poon<br>10:30 - 11:00 – Seung-Hyeok Kye<br>11:00 - 11:30 – Shmuel Friedland<br>11:30 - 12:00 – Xiaofei Qi |
| **Tuesday, 10:00AM - 12:00PM**<br>**Location: Bristol A** | **Randomized Matrix Algorithms**<br>10:00 - 10:30 – Ilse Ipsen<br>10:30 - 11:00 – Michael Mahoney<br>11:00 - 11:30 – Haim Avron<br>11:30 - 12:00 – Alex Gittens |
| **Tuesday, 10:00AM - 12:00PM**<br>**Location: Bristol B** | **Linear Algebra, Control, and Optimization**<br>10:00 - 10:30 – Biswa Datta<br>10:30 - 11:00 – Paul Van Dooren<br>11:00 - 11:30 – Melvin Leok<br>11:30 - 12:00 – Volker Mehrmann |
| **Tuesday, 10:00AM - 12:00PM**<br>**Location: Tiverton** | **Contributed Session on Computational Science**<br>10:00 - 10:20 – Youngmi Hur<br>10:20 - 10:40 – Margarida Mitjana<br>10:40 - 11:00 – Matthias Bolten<br>11:00 - 11:20 – Hashim Saber<br>11:20 - 11:40 – Angeles Carmona<br>11:40 - 12:00 – Andrey Melnikov |
| **Tuesday, 10:00AM - 12:00PM**<br>**Location: Ocean** | **Matrix Methods for Polynomial Root-Finding**<br>10:00 - 10:30 – David Watkins<br>10:30 - 11:00 – Jared Aurentz<br>11:00 - 11:30 – Yuli Eidelman<br>11:30 - 12:00 – Matthias Humet |
| **Tuesday, 10:00AM - 12:00PM**<br>**Location: Patriots** | **Multilinear Algebra and Tensor Decompositions**<br>10:00 - 10:30 – Charles Van Loan<br>10:30 - 11:00 – Martin Mohlenkamp<br>11:00 - 11:30 – Manda Winlaw<br>11:30 - 12:00 – Eugene Tyrtyshnikov |

| | |
|---|---|
| **Tuesday, 12:00PM - 01:20PM**<br>**Location: Grand Foyer** | **Lunch - Attendees on their own - Lunch on sale in Grand Foyer** |

| | |
|---|---|
| **Tuesday, 01:20PM - 02:05PM**<br>**Location: Grand Ballroom** | **Ivan Oseledets (Institute of Numerical Mathematics, RAS)**<br>Session Chair: Misha Kilmer |

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|---|---|
| **Tuesday, 02:15PM - 04:15PM**<br>**Location: Grand Ballroom** | **Matrices and Graph Theory**<br>2:15 - 2:45 – Minerva Catral<br>2:45 - 3:15 – Seth Meyer<br>3:15 - 3:45 – Adam Berliner<br>3:45 - 4:15 – Michael Young |
| **Tuesday, 02:15PM - 04:15PM**<br>**Location: Bristol A** | **Linear Complementarity Problems and Beyond**<br>2:15 - 2:45 – Richard Cottle<br>2:45 - 3:15 – Ilan Adler<br>3:15 - 3:45 – Jinglai Shen<br>3:45 - 4:15 – Youngdae Kim |
| **Tuesday, 02:15PM - 04:15PM**<br>**Location: Bristol B** | **Linear Algebra, Control, and Optimization**<br>2:15 - 2:45 – Michael Overton<br>2:45 - 3:15 – Shankar Bhattacharyya<br>3:15 - 3:45 – Melina Freitag<br>3:45 - 4:15 – Bart Vandereycken |
| **Tuesday, 02:15PM - 04:15PM**<br>**Location: Tiverton** | **Contributed Session on Numerical Linear Algebra Part 1**<br>2:15 - 2:35 – Marko Petkovic<br>2:35 - 2:55 – Jesse Barlow<br>2:55 - 3:15 – James Lambers<br>3:15 - 3:35 – Froilán Dopico |
| **Tuesday, 02:15PM - 04:15PM**<br>**Location: Ocean** | **Matrix Methods for Polynomial Root-Finding**<br>2:15 - 2:45 – Victor Pan<br>2:45 - 3:15 – Peter Strobach<br>3:15 - 3:45 – Leonardo Robol<br>3:45 - 4:15 – Raf Vandebril |
| **Tuesday, 02:15PM - 04:15PM**<br>**Location: Patriots** | **Multilinear Algebra and Tensor Decompositions**<br>2:15 - 2:45 – Shmuel Friedland<br>2:45 - 3:15 – Andre Uschmajew<br>3:15 - 3:45 – Sergey Dolgov<br>3:45 - 4:15 – Sergey Dolgov |

| | |
|---|---|
| **Tuesday, 04:15PM - 04:45PM**<br>**Location: Grand Foyer** | **Coffee Break** |

| | |
|---|---|
| **Tuesday, 04:45PM - 06:45PM**<br>**Location: Grand Ballroom** | **Matrices and Graph Theory**<br>4:45 - 5:15 – Woong Kook<br>5:15 - 5:45 – Bryan Shader<br>5:45 - 6:15 – Louis Deaett<br>6:15 - 6:45 – Colin Garnett |
| **Tuesday, 04:45PM - 06:45PM**<br>**Location: Bristol A** | **Applications of Tropical Mathematics**<br>4:45 - 5:15 – Meisam Sharify<br>5:15 - 5:45 – James Hook<br>5:45 - 6:15 – Andrea Marchesini<br>6:15 - 6:45 – Nikolai Krivulin |
| **Tuesday, 04:45PM - 06:45PM**<br>**Location: Bristol B** | **Contributed Session on Numerical Linear Algebra Part 2**<br>4:45 - 5:05 – Clemente Cesarano<br>5:05 - 5:25 – Surya Prasath<br>5:25 - 5:45 – Na Li<br>5:45 - 6:05 – Yusaku Yamamoto<br>6:05 - 6:25 – Luis Verde-Star |
| **Tuesday, 04:45PM - 06:45PM**<br>**Location: Ocean** | **Matrix Methods for Polynomial Root-Finding**<br>4:45 - 5:15 – Luca Gemignani<br>5:15 - 5:45 – Gianna Del Corso<br>5:45 - 6:15 – Olga Holtz |
| **Tuesday, 04:45PM - 06:45PM**<br>**Location: Patriots** | **Multilinear Algebra and Tensor Decompositions**<br>4:45 - 5:15 – Vladimir Kazeev<br>5:15 - 5:45 – Misha Kilmer<br>5:45 - 6:15 – Lek-Heng Lim<br>6:15 - 6:45 – Lieven De Lathauwer |

# Wednesday

| | |
|---|---|
| **Wednesday, 08:00AM - 08:45AM**<br>**Location: Grand Ballroom** | **Thomas Laffey (University College Dublin) (Hans Schneider prize winner)**<br>Session Chair: Shmuel Friedland |
| **Wednesday, 08:45AM - 09:30AM**<br>**Location: Grand Ballroom** | **Gilbert Strang (MIT)**<br>Session Chair: Shmuel Friedland |

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| **Wednesday, 09:30AM - 10:00AM**<br>**Location: Grand Foyer** | **Coffee Break** |

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| **Wednesday, 10:00AM - 12:00PM**<br>**Location: Bristol A** | **Matrices and Orthogonal Polynomials**<br>10:00 - 10:30 – Paul Terwilliger<br>10:30 - 11:00 – Edward Hanson<br>11:00 - 11:30 – Antonio Duran<br>11:30 - 12:00 – Holger Dette |
| **Wednesday, 10:00AM - 12:00PM**<br>**Location: Bristol B** | **Symbolic Matrix Algorithms**<br>10:00 - 10:30 – Gilles Villard<br>10:30 - 11:00 – Arne Storjohann<br>11:00 - 11:30 – Wayne Eberly<br>11:30 - 12:00 – David Saunders |
| **Wednesday, 10:00AM - 12:00PM**<br>**Location: Tiverton** | **Contributed Session on Numerical Range and Spectral Problems**<br>10:00 - 10:20 – Christopher Bailey<br>10:20 - 10:40 – Yuri Nesterenko<br>10:40 - 11:00 – Michael Karow<br>11:00 - 11:20 – Xuhua Liu |
| **Wednesday, 10:00AM - 12:00PM**<br>**Location: Ocean** | **Matrices and Graph Theory**<br>10:00 - 10:30 – Pauline van den Driessche<br>10:30 - 11:00 – Oliver Knill<br>11:00 - 11:30 – Steve Kirkland<br>11:30 - 12:00 – Jason Molitierno |
| **Wednesday, 10:00AM - 12:00PM**<br>**Location: Patriots** | **Structure and Randomization in Matrix Computations**<br>10:00 - 10:30 – Victor Pan<br>10:30 - 11:00 – Yousef Saad<br>11:00 - 11:30 – Paul Van Dooren<br>11:30 - 12:00 – Francoise Tisseur |

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| **Wednesday, 12:00PM - 01:00PM**<br>**Location: Grand Foyer** | **Lunch - Attendees on their own - Lunch on sale in Grand Foyer** |

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| **Wednesday, 01:00PM - 10:00PM**<br>**Location: Newport, RI** | **Excursion - Newport Mansion and Dinner in Downtown Newport** |

# Thursday

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| **Thursday, 08:00AM - 08:45AM**<br>**Location: Grand Ballroom** | **Fuzhen Zhang (Nova Southeastern University)**<br>Session Chair: Vadim Olshevsky |
| **Thursday, 08:45AM - 09:30AM**<br>**Location: Grand Ballroom** | **Dianne O'Leary (University of Maryland)**<br>Session Chair: Vadim Olshevsky |

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| **Thursday, 09:30AM - 10:00AM**<br>**Location: Grand Foyer** | **Coffee Break** |

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|---|---|
| **Thursday, 10:00AM - 12:00PM**<br>**Location: Grand Ballroom** | **Matrices and Orthogonal Polynomials**<br>10:00 - 10:30 – Luc Vinet<br>10:30 - 11:00 – Jeff Geronimo<br>11:00 - 11:30 – Manuel Manas<br>11:30 - 12:00 – Hugo Woerdeman |
| **Thursday, 10:00AM - 12:00PM**<br>**Location: Bristol A** | **Contributed Session on Numerical Linear Algebra Part 3**<br>10:00 - 10:20 – Kim Batselier<br>10:20 - 10:40 – Xuzhou Chen<br>10:40 - 11:00 – William Morrow<br>11:00 - 11:20 – Pedro Freitas<br>11:20 - 11:40 – Abderrahman Bouhamidi<br>11:40 - 12:00 – Roel Van Beeumen |
| **Thursday, 10:00AM - 12:00PM**<br>**Location: Bristol B** | **Symbolic Matrix Algorithms**<br>10:00 - 10:30 – Victor Pan<br>10:30 - 11:00 – Brice Boyer<br>11:00 - 11:30 – Erich Kaltofen<br>11:30 - 12:00 – George Labahn |
| **Thursday, 10:00AM - 12:00PM**<br>**Location: Tiverton** | **Contributed Session on Matrix Pencils**<br>10:00 - 10:20 – Javier Perez<br>10:20 - 10:40 – Maria Isabel Bueno Cachadina<br>10:40 - 11:00 – Michal Wojtylak<br>11:00 - 11:20 – Andrii Dmytryshyn<br>11:20 - 11:40 – Vasilije Perovic<br>11:40 - 12:00 – Susana Furtado |
| **Thursday, 10:00AM - 12:00PM**<br>**Location: Ocean** | **Matrices and Graph Theory**<br>10:00 - 10:30 – Steven Osborne<br>10:30 - 11:00 – Steve Butler<br>11:00 - 11:30 – T. S. Michael<br>11:30 - 12:00 – Shaun Fallat |
| **Thursday, 10:00AM - 12:00PM**<br>**Location: Patriots** | **Structure and Randomization in Matrix Computations**<br>10:00 - 10:30 – Yuli Eidelman<br>10:30 - 11:00 – Luca Gemignani<br>11:00 - 11:30 – Marc Baboulin<br>11:30 - 12:00 – Thomas Mach |

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| **Thursday, 12:00PM - 01:30PM**<br>**Location: Grand Foyer** | **Lunch - Attendees on their own - Lunch on sale in Grand Foyer** |

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| **Thursday, 01:30PM - 03:30PM**<br>**Location: Grand Ballroom** | **Matrices and Orthogonal Polynomials**<br>1:30 - 2:00 – Luis Velazquez<br>2:00 - 2:30 – Francisco Marcellan<br>2:30 - 3:00 – Marko Huhtanen<br>3:00 - 3:30 – Rostyslav Kozhan |
| **Thursday, 01:30PM - 03:30PM**<br>**Location: Bristol A** | **Linear and Nonlinear Perron-Frobenius Theory**<br>1:30 - 2:00 – Roger Nussbaum<br>2:00 - 2:30 – Helena Šmigoc<br>2:30 - 3:00 – Julio Moro<br>3:00 - 3:30 – Yongdo Lim |
| **Thursday, 01:30PM - 03:30PM**<br>**Location: Bristol B** | **Linear Algebra Education Issues**<br>1:30 - 2:00 – Gilbert Strang<br>2:00 - 2:30 – Judi McDonald<br>2:30 - 3:00 – Sandra Kingan<br>3:00 - 3:30 – Jeffrey Stuart |
| **Thursday, 01:30PM - 03:30PM**<br>**Location: Tiverton** | **Contributed Session on Nonnegative Matrices Part 1**<br>1:30 - 1:50 – Keith Hooper<br>1:50 - 2:10 – Akiko Fukuda<br>2:10 - 2:30 – Anthony Cronin<br>2:30 - 2:50 – Shani Jose<br>2:50 - 3:10 – Prashant Batra<br>3:10 - 3:30 – Pietro Paparella |
| **Thursday, 01:30PM - 03:30PM**<br>**Location: Ocean** | **Contributed Session on Graph Theory**<br>1:30 - 1:50 – Caitlin Phifer<br>1:50 - 2:10 – Milica Andelic<br>2:10 - 2:30 – David Jacobs<br>2:30 - 2:50 – Nathan Warnberg<br>2:50 - 3:10 – Andres Encinas<br>3:10 - 3:30 – Joseph Fehribach |
| **Thursday, 01:30PM - 03:30PM**<br>**Location: Patriots** | **Structure and Randomization in Matrix Computations**<br>1:30 - 2:00 – Jianlin Xia<br>2:00 - 2:30 – Ilse Ipsen<br>2:30 - 3:00 – Victor Pan<br>3:00 - 3:30 – Michael Mahoney |

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| **Thursday, 03:30PM - 05:30PM**<br>**Location: Grand Ballroom** | **ILAS Membership Business Meeting** |

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| **Thursday, 05:30PM - 07:30PM**<br>**Location: Grand Ballroom** | **Linear and Nonlinear Perron-Frobenius Theory**<br>5:30 - 6:00 – Marianne Akian<br>6:00 - 6:30 – Assaf Goldberger<br>6:30 - 7:00 – Brian Lins<br>7:00 - 7:30 – Olga Kushel |
| **Thursday, 05:30PM - 07:30PM**<br>**Location: Bristol A** | **Matrix Inequalities**<br>5:30 - 6:00 – Fuzhen Zhang<br>6:00 - 6:30 – Takashi Sano<br>6:30 - 7:00 – Charles Johnson<br>7:00 - 7:30 – Tomohiro Hayashi |
| **Thursday, 05:30PM - 07:30PM**<br>**Location: Bristol B** | **Structured Matrix Functions and their Applications (Dedicated to Leonia Lerer on the occasion of his 70th birthday)**<br>5:30 - 6:00 – Leiba Rodman<br>6:00 - 6:30 – Gilbert Strang<br>6:30 - 7:00 – Marinus Kaashoek |
| **Thursday, 05:30PM - 07:30PM**<br>**Location: Tiverton** | **Contributed Session on Matrix Completion Problems**<br>5:30 - 5:50 – James McTigue<br>5:50 - 6:10 – Gloria Cravo<br>6:10 - 6:30 – Zheng QU<br>6:30 - 6:50 – Ryan Wasson<br>6:50 - 7:10 – Yue Liu |
| **Thursday, 05:30PM - 07:30PM**<br>**Location: Ocean** | **Abstract Interpolation and Linear Algebra**<br>5:30 - 6:00 – Sanne ter Horst<br>6:00 - 6:30 – Christopher Beattie<br>6:30 - 7:00 – Johan Karlsson<br>7:00 - 7:30 – Dan Volok |
| **Thursday, 05:30PM - 07:30PM**<br>**Location: Patriots** | **Sign Pattern Matrices**<br>5:30 - 6:00 – Zhongshan Li<br>6:00 - 6:30 – Judi McDonald<br>6:30 - 7:00 – Leslie Hogben<br>7:00 - 7:30 – Craig Erickson |

# Friday

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| **Friday, 08:00AM - 08:45AM**<br>**Location: Grand Ballroom** | **Dan Spielman (Yale University)**<br>Session Chair: Froilán Dopico |
| **Friday, 08:45AM - 09:30AM**<br>**Location: Grand Ballroom** | **Jean Bernard Lasserre (Centre National de la Recherche Scientifique, France)**<br>Session Chair: Froilán Dopico |

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| **Friday, 09:30AM - 10:00AM**<br>**Location: Grand Foyer** | **Coffee Break** |

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| **Friday, 10:00AM - 12:00PM**<br>**Location: Grand Ballroom** | **Matrices and Orthogonal Polynomials**<br>10:00 - 10:30 – Carl Jagels<br>10:30 - 11:00 – Vladimir Druskin<br>11:00 - 11:30 – Lothar Reichel<br>11:30 - 12:00 – Raf Vandebril |
| **Friday, 10:00AM - 12:00PM**<br>**Location: Bristol A** | **Matrix Inequalities**<br>10:00 - 10:30 – Tin-Yau Tam<br>10:30 - 11:00 – Fuad Kittaneh<br>11:00 - 11:30 – Rajesh Pereira<br>11:30 - 12:00 – Ameur Seddik |
| **Friday, 10:00AM - 12:00PM**<br>**Location: Bristol B** | **Linear Algebra Education Issues**<br>10:00 - 10:30 – David Strong<br>10:30 - 11:00 – Rachel Quinlan<br>11:00 - 11:30 – Megan Wawro<br>11:30 - 12:00 – Sepideh Stewart |
| **Friday, 10:00AM - 12:00PM**<br>**Location: Tiverton** | **Contributed Session on Nonnegative Matrices Part 2**<br>10:00 - 10:20 – James Weaver<br>10:20 - 10:40 – Hiroshi Kurata<br>10:40 - 11:00 – Maguy Trefois<br>11:00 - 11:20 – Ravindra Bapat<br>11:20 - 11:40 – Naomi Shaked-Monderer |
| **Friday, 10:00AM - 12:00PM**<br>**Location: Ocean** | **Abstract Interpolation and Linear Algebra**<br>10:00 - 10:30 – Izchak Lewkowicz<br>10:30 - 11:00 – Quanlei Fang<br>11:00 - 11:30 – Daniel Alpay<br>11:30 - 12:00 – Vladimir Bolotnikov |
| **Friday, 10:00AM - 12:00PM**<br>**Location: Patriots** | **Sign Pattern Matrices**<br>10:00 - 10:30 – Marina Arav<br>10:30 - 11:00 – Wei Gao<br>11:00 - 11:30 – Timothy Melvin |

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| **Friday, 12:00PM - 01:20PM**<br>**Location: Grand Foyer** | **Lunch - Attendees on their own - Lunch on sale in Grand Foyer** |

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| **Friday, 01:20PM - 02:05PM**<br>**Location: Grand Ballroom** | **Ravi Kannan (Microsoft Research)**<br>Session Chair: Jianlin Xia |

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| **Friday, 02:15PM - 04:15PM**<br>**Location: Grand Ballroom** | **Structured Matrix Functions and their Applications (Dedicated to Leonia Lerer on the occasion of his 70th birthday)**<br>2:15 - 2:45 – Joseph Ball<br>2:45 - 3:15 – Alexander Sakhnovich<br>3:15 - 3:45 – Hermann Rabe<br>3:45 - 4:15 – Harm Bart |
| **Friday, 02:15PM - 04:15PM**<br>**Location: Bristol A** | **Matrices and Total Positivity**<br>2:15 - 2:45 – Charles Johnson<br>2:45 - 3:15 – Gianna Del Corso<br>3:15 - 3:45 – Olga Kushel<br>3:45 - 4:15 – Rafael Canto |
| **Friday, 02:15PM - 04:15PM**<br>**Location: Bristol B** | **Linear Algebra Education Issues**<br>2:15 - 2:45 – Robert Beezer<br>2:45 - 3:15 – Tom Edgar<br>3:15 - 3:45 – Sang-Gu Lee<br>3:45 - 4:15 – Jason Grout |
| **Friday, 02:15PM - 04:15PM**<br>**Location: Ocean** | **Matrix Inequalities**<br>2:15 - 2:45 – Natalia Bebiano<br>2:45 - 3:15 – Takeaki Yamazaki<br>3:15 - 3:45 – Minghua Lin |
| **Friday, 02:15PM - 04:15PM**<br>**Location: Patriots** | **Contributed Session on Positive Definite Matrices**<br>2:15 - 2:35 – Bala Rajaratnam<br>2:35 - 2:55 – Apoorva Khare<br>2:55 - 3:15 – Dominique Guillot |

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| **Friday, 04:15PM - 04:45PM**<br>**Location: Grand Foyer** | **Coffee Break** |

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| **Friday, 04:45PM - 06:45PM**<br>**Location: Grand Ballroom** | **Structured Matrix Functions and their Applications (Dedicated to Leonia Lerer on the occasion of his 70th birthday)**<br>4:45 - 5:15 – Ilya Spitkovsky<br>5:15 - 5:45 – David Kimsey<br>5:45 - 6:15 – Hugo Woerdeman |
| **Friday, 04:45PM - 06:45PM**<br>**Location: Bristol A** | **Matrices and Total Positivity**<br>4:45 - 5:15 – Jorge Delgado<br>5:15 - 5:45 – Alvaro Barreras<br>5:45 - 6:15 – José-Javier Martínez<br>6:15 - 6:45 – Stephane Launois |
| **Friday, 04:45PM - 06:45PM**<br>**Location: Bristol B** | **Matrix Methods in Computational Systems Biology and Medicine**<br>4:45 - 5:15 – Lee Altenberg<br>5:15 - 5:45 – Natasa Durdevac<br>5:45 - 6:15 – Marcus Weber<br>6:15 - 6:45 – Amir Niknejad |
| **Friday, 04:45PM - 06:45PM**<br>**Location: Tiverton** | **Contributed Session on Matrix Equalities and Inequalities**<br>4:45 - 5:05 – Bas Lemmens<br>5:05 - 5:25 – Dawid Janse van Rensburg<br>5:25 - 5:45 – Hosoo Lee<br>5:45 - 6:05 – Jadranka Micic Hot |
| **Friday, 04:45PM - 06:45PM**<br>**Location: Ocean** | **Linear Algebra Problems in Quantum Computation**<br>4:45 - 5:15 – Man-Duen Choi<br>5:15 - 5:45 – Panayiotis Psarrakos<br>5:45 - 6:15 – Raymond Nung-Sing Sze |
| **Friday, 04:45PM - 06:45PM**<br>**Location: Patriots** | **Applications of Tropical Mathematics**<br>4:45 - 5:15 – Marianne Johnson<br>5:15 - 5:45 – Pascal Benchimol<br>5:45 - 6:15 – Marie MacCaig<br>6:15 - 6:45 – Viorel Nitica |

## Monday, 08:00AM - 08:45AM

| **Anne Greenbaum** | **Location: Grand Ballroom** |
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| **$K$-Spectral Sets and Applications** | |

Let $A$ be an $n$ by $n$ matrix or a bounded linear operator on a Hilbert space. If $f : \mathbb{C} \to \mathbb{C}$ is a function that is holomorphic in a region $\Omega$ of the complex plane containing a neighborhood of the spectrum of $A$, then the matrix or operator $f(A)$ can be defined through the Cauchy integral formula. For many years, there has been interest in relating the 2-norm of the operator $f(A)$ (that is, $\|f(A)\| \equiv \sup_{\|v\|_2=1} \|f(A)v\|_2$) to the $\mathcal{L}^\infty$-norm of $f$ on such a set $\Omega$ (that is, $\|f\|_\Omega \equiv \sup_{z\in\Omega} |f(z)|$). If all rational functions $f$ with poles outside $\Omega$ satisfy $\|f(A)\| \leqslant K\|f\|_\Omega$, then $\Omega$ is said to be a *$K$-spectral set* for $A$.

The first identification of a 1-spectral set (or, just a *spectral set*) was by von Neumann in 1951: If $A$ is a contraction (that is, $\|A\| \leqslant 1$) then $\|f(A)\| \leqslant \|f\|_{\mathcal{D}}$ where $\mathcal{D}$ denotes the unit disk. More recently (2004), M. Crouzeix has shown that the *field of values* or *numerical range* ($W(A) \equiv \{\langle Aq, q\rangle : \|q\|_2 = 1\}$) is a $K$-spectral set for $A$ with $K = 11.08$, and he conjectures that $K$ can be taken to be 2. Another $K$-spectral set is the $\epsilon$-pseudospectrum of $A$: $\Lambda_\epsilon(A) \equiv \{z \in \mathbb{C} : \|(zI - A)^{-1}\| > \epsilon^{-1}\}$. If $L$ denotes the length of the boundary of $\Lambda_\epsilon(A)$, then $\|f(A)\| \leqslant (L/(2\pi\epsilon))\|f\|_{\Lambda_\epsilon(A)}$. Many other $K$-spectral sets can be derived for a given operator $A$ by noting that if $\Omega$ is a $K$-spectral set for $B$ and if $A = f(B)$, then $f(\Omega)$ is a $K$-spectral set for $A$.

In this talk, I will discuss these ideas and others, concentrating on the conjecture of Crouzeix that the field of values of $A$ is a 2-spectral set for $A$. A (possibly) stronger conjecture is that if $g$ is a biholomorphic mapping from $W(A)$ onto $\mathcal{D}$, then $g(A)$ is similar to a contraction via a similarity transformation with condition number at most 2; that is, $g(A) = XCX^{-1}$, where $\|C\| \leqslant 1$ and $\kappa(X) \equiv \|X\| \cdot \|X^{-1}\| \leqslant 2$. An advantage of this statement is that it can be checked numerically (and perhaps even proved through careful arguments about the accuracy of the computation) for specific matrices $A$. I will discuss implications of these estimates and conjectures for various problems of applied and computational mathematics, in particular, what they can tell us about the convergence rate of the GMRES algorithm for solving linear systems. In some cases, good bounds can be obtained even when the origin lies inside the field of values by using the fact that if $A = f(B)$ then $f(W(B))$ is a $K$-spectral set for $A$.

## Monday, 08:45AM - 09:30AM

| **Leiba Rodman** | **Location: Grand Ballroom** |
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| **Eigenvalue perturbation analysis of structured matrices** | |

We study the eigenvalue perturbation theory of structured matrices under structured rank one perturbation which may or may not be small in norm. Structured matrices in question are defined by various symmetry properties, such as selfadjoint or symplectic with respect to a sesquilinear, symmetric bilinear, or skew-symmetric form, over the real or complex numbers. Generic Jordan structures of the perturbed matrix are identified in many cases. The talk is based on joint work with C. Mehl, V. Mehrmann, and A.C.M. Ran.

## Monday, 10:00AM - 12:00PM

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| **Linear Algebra Problems in Quantum Computation** | | **Grand Ballroom** |
| 10:00 - 10:30 | Jinchuan Hou | Quantum measurements and maps preserving strict convex combinations and pure states |
| 10:30 - 11:00 | Lajos Molnar | Some non-linear preservers on quantum structures |
| 11:00 - 11:30 | Zejun Huang | Linear preserver problems arising in quantum information science |
| 11:30 - 12:00 | Gergo Nagy | Transformations on density matrices preserving the Holevo bound |
| **Advances in Combinatorial Matrix Theory and its Applications** | | **Bristol A** |
| 10:00 - 10:30 | Carlos da Fonseca | On the P-sets of acyclic matrices |
| 10:30 - 11:00 | Ferenc Szollosi | Large set of equiangular lines in Euclidean spaces |
| 11:00 - 11:30 | Geir Dahl | Majorization transforms and Ryser's algorithm |
| 11:30 - 12:00 | Judi McDonald | Good Bases for Nonnegative Realizations |
| **Generalized Inverses and Applications** | | **Bristol B** |
| 10:00 - 10:30 | Minnie Catral | Matrix groups constructed from $\{R, s+1, k\}$-potent matrices |
| 10:30 - 11:00 | Nieves Castro-Gonzalez | Generalized inverses and extensions of the Sherman-Morrison-Woodbury formulae |
| 11:00 - 11:30 | Esther Dopazo | Formulas for the Drazin inverse of block anti-triangular matrices |
| **Nonlinear Eigenvalue Problems** | | **Ocean** |
| 10:00 - 10:30 | Francoise Tisseur | A Review of Nonlinear Eigenvalue Problems |
| 10:30 - 11:00 | Karl Meerbergen | An overview of Krylov methods for nonlinear eigenvalue problems |
| 11:00 - 11:30 | Elias Jarlebring | Block iterations for eigenvector nonlinearities |
| 11:30 - 12:00 | Zhaojun Bai | A Padé Approximate Linearization Technique for Solving the Quadratic Eigenvalue Problem with Low-Rank Damping |
| **Linear Least Squares Methods: Algorithms, Analysis, and Applications** | | **Patriots** |
| 10:00 - 10:30 | David Titley-Peloquin | Stochastic conditioning of systems of equations |
| 10:30 - 11:00 | Huaian Diao | Structured Condition Numbers for a Linear Functional of the Tikhonov Regularized Solution |
| 11:00 - 11:30 | Sanzheng Qiao | Lattice Basis Reduction: Preprocessing the Integer Least Squares Problem |
| 11:30 - 12:00 | Keiichi Morikuni | Inner-iteration preconditioning for the minimum-norm solution of rank-deficient linear systems |

## Monday, 01:20PM - 02:05PM

**Alan Edelman** — **Location: Grand Ballroom**

**The Interplay of Random Matrix Theory, Numerical Linear Algebra, and Ghosts and Shadows**

Back in the day, numerical linear algebra was about the construction of SVDs, GSVDs, tridiagonals, bidiagonals, arrow matrices, Householder Reflectors, Givens rotations and so much more. In this talk, we will show the link to classical finite random matrix theory. In particular, we show how the numerical algorithms of yesteryear are playing a role today that we do not completely understand. What is happening is that our favorite algorithms seem to be working not only on reals, complexes, and quaternions, but on spaces of any arbitrary dimension, at least in a very simple formal sense. A continuing theme is that the algorithms from numerical linear algebra seem as though they would be incredibly useful even if there were never any computers to run them on.

## Monday, 02:15PM - 04:15PM

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| **Linear Algebra Problems in Quantum Computation** | | **Grand Ballroom** |
| 2:15 - 2:45 | David Kribs | Private quantum subsystems and connections with quantum error correction |
| 2:45 - 3:15 | Jianxin Chen | Rank reduction for the local consistency problem |
| 3:15 - 3:45 | Nathaniel Johnston | On the Minimum Size of Unextendible Product Bases |
| 3:45 - 4:15 | Sarah Plosker | On trumping |
| **Advances in Combinatorial Matrix Theory and its Applications** | | **Bristol A** |
| 2:15 - 2:45 | Gi-Sang Cheon | Several types of Bessel numbers generalized and some combinatorial setting |
| 2:45 - 3:15 | Kathleen Kiernan | Patterns of Alternating Sign Matrices |
| 3:15 - 3:45 | Kevin Vander Meulen | Companion Matrices and Spectrally Arbitrary Patterns |
| 3:45 - 4:15 | Ryan Tifenbach | A combinatorial approach to nearly uncoupled Markov chains |
| **Generalized Inverses and Applications** | | **Bristol B** |
| 2:15 - 2:45 | Huaian Diao | On the Level-2 Condition Number for MooreCPenrose Inverse in Hilbert Space |
| 2:45 - 3:15 | Chunyuan Deng | On invertibility of combinations of $k$-potent operators |
| 3:15 - 3:45 | Dragana Cvetkovic-Ilic | Reverse order law for the generalized inverses |
| 3:45 - 4:15 | Dijana Mosic | Representations for the generalized Drazin inverse of block matrices in Banach algebras |
| **Contributed Session on Algebra and Matrices over Arbitrary Fields Part 1** | | **Tiverton** |
| 2:15 - 2:35 | Polona Oblak | On extremal matrix centralizers |
| 2:35 - 2:55 | Gregor Dolinar | Commutativity preserving maps via maximal centralizers |
| 3:15 - 3:35 | Leo Livshits | Paratransitive algebras of linear transformations and a spatial implementation of “Wedderburn’s Principal Theorem” |
| 3:35 - 3:55 | Peter Semrl | Adjacency preserving maps |
| **Nonlinear Eigenvalue Problems** | | **Ocean** |
| 2:15 - 2:45 | Ion Zaballa | The Inverse Symmetric Quadratic Eigenvalue Problem |
| 2:45 - 3:15 | Shreemayee Bora | Distance problems for Hermitian matrix polynomials |
| 3:15 - 3:45 | Leo Taslaman | Exploiting low rank of damping matrices using the Ehrlich-Aberth method |
| 3:45 - 4:15 | Christian Mehl | Skew-symmetry - a powerful structure for matrix polynomials |
| **Krylov Subspace Methods for Linear Systems** | | **Patriots** |
| 2:15 - 2:45 | Eric de Sturler | The convergence behavior of BiCG |
| 2:45 - 3:15 | Daniel Richmond | Implicitly Restarting the LSQR Algorithm |
| 3:15 - 3:45 | Andreas Stathopoulos | Using ILU(0) to estimate the diagonal of the inverse of a matrix. |
| 3:45 - 4:15 | Daniel Szyld | MPGMRES: a generalized minimum residual method with multiple preconditioners |

## Monday, 04:45PM - 06:45PM

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| **Linear Algebra Problems in Quantum Computation** | | **Grand Ballroom** |
| 4:45 - 5:15 | Tin-Yau Tam | Gradient Flows for the Minimum Distance to the Sum of Adjoint Orbits |
| 5:15 - 5:45 | Diane Christine Pelejo | Decomposing a Unitary Matrix into a Product of Weighted Quantum Gates |
| 5:45 - 6:15 | Justyna Zwolak | Higher dimensional witnesses for entanglement in $2N$–qubit chains |
| **Advances in Combinatorial Matrix Theory and its Applications** | | **Bristol A** |
| 4:45 - 5:15 | In-Jae Kim | Creating a user-satisfaction index |
| 5:15 - 5:45 | Charles Johnson | Eigenvalues, Multiplicities and Linear Trees |
| **Matrices over Idempotent Semirings** | | **Bristol B** |
| 4:45 - 5:15 | Sergei Sergeev | Gaussian elimination and other universal algorithms |
| 5:15 - 5:45 | Jan Plavka | The robustness of interval matrices in max-min algebra |
| 5:45 - 6:15 | Hans Schneider | Bounds of Wielandt and Schwartz for max-algebraic matrix powers |
| 6:15 - 6:45 | Martin Gavalec | Tolerance and strong tolerance interval eigenvectors in max-min algebra |
| **Contributed Session on Algebra and Matrices over Arbitrary Fields Part 2** | | **Tiverton** |
| 4:45 - 5:05 | Mikhail Stroev | Permanents, determinants, and generalized complementary basic matrices |
| 5:05 - 5:25 | Bart De Bruyn | On trivectors and hyperplanes of symplectic dual polar spaces |
| 5:25 - 5:45 | Rachel Quinlan | Partial matrices of constant rank over small fields |
| 5:45 - 6:05 | Yorick Hardy | Uniform Kronecker Quotients |
| **Nonlinear Eigenvalue Problems** | | **Ocean** |
| 4:45 - 5:15 | D. Steven Mackey | A Filtration Perspective on Minimal Indices |
| 5:15 - 5:45 | Fernando De Teran | Spectral equivalence of Matrix Polynomials, the Index Sum Theorem and consequences |
| 5:45 - 6:15 | Vanni Noferini | Vector spaces of linearizations for matrix polynomials: a bivariate polynomial approach |
| 6:15 - 6:45 | Federico Poloni | Duality of matrix pencils, singular pencils and linearizations |
| **Krylov Subspace Methods for Linear Systems** | | **Patriots** |
| 4:45 - 5:15 | James Baglama | Restarted Block Lanczos Bidiagonalization Algorithm for Finding Singular Triplets |
| 5:15 - 5:45 | Jennifer Pestana | GMRES convergence bounds that depend on the right-hand side vector |
| 5:45 - 6:15 | Xiaoye Li | Robust and Scalable Schur Complement Method using Hierarchical Parallelism |

## Tuesday, 08:00AM - 08:45AM

**Raymond Sze** **Location: Grand Ballroom**
**Quantum Operation and Quantum Error Correction**

Quantum operation is one of the most fundamental and important concepts which is widely used in all aspects of quantum information theory. In the context of quantum computation, a quantum operation is also called a quantum channel. A quantum channel can be viewed as a communication channel in a quantum system which can transmit quantum information such as qubits. Mathematically, a quantum channel is a trace preserving completely positive map between quantum state spaces with the operator sum representation $\rho \mapsto \sum_{j=1}^{r} E_j \rho E_j^*$ with $\sum_{j=1}^{r} E_j^* E_j = I$.

One of the fundamental problems quantum information scientists concerned with, is whether one can design and construct a quantum device that transforms certain quantum states into other quantum states. This task is physically possible if a specified quantum operation (transformation) of certain prescribed sets of input and output states can be found. The problem then becomes to determine an existence condition of a trace preserving completely positive map sending $\rho_j$ to $\sigma_j$ for all $j$, for certain given sets of quantum states $\{\rho_1, \ldots, \rho_k\}$ and $\{\sigma_1, \ldots, \sigma_k\}$. This is called the problem of state transformation. In this talk, recent results on this problem will be presented.

When a quantum system interacts with the outside world, it will be vulnerable to disturbance from the external environment which can lead to decoherence in the system. Decoherence can be regarded as a quantum operation in which environment causes undesirable noise in the quantum system. Quantum error correction is one of the strategies to flight against decoherence. In this talk, we review different error correction schemes including decoherence-free subspace model, noiseless subsystem model, and operator quantum error correction model, in an operator theoretical approach.

## Tuesday, 08:45AM - 09:30AM

**Maryam Fazel** **Location: Grand Ballroom**
**Recovery of Structured Models with Limited Information**

Finding models with a low-dimensional structure given a limited number of observations is a central problem in signal processing (compressed sensing), machine learning (recommender systems), and system identification. Examples of such structured models include sparse vectors, low-rank matrices, and the sum of sparse and low-rank matrices. We begin by reviewing recent results for these structures, characterizing the number of observations for successful model recovery.

We then examine a new problem: In many signal processing and machine learning applications, the desired model has *multiple* structures simultaneously. Applications include sparse phase retrieval, and learning models with several structural priors in machine learning tasks. Often, penalties that promote individual structures are known, and require a minimal number of generic measurements (e.g.,$\ell_1$ norm for sparsity, nuclear norm for matrix rank), so it is reasonable to minimize a combination of such norms. We show that, surprisingly, if we use multiobjective optimization with the individual norms, we can do no better (order-wise) than an algorithm that exploits only one of the structures. This result holds in a general setting and suggests that to fully exploit the multiple structures, we need an entirely new convex relaxation. We also discuss the related problem of 'denoising' for simultaneously structured signals corrupted by additive noise.

## Tuesday, 10:00AM - 12:00PM

| | | |
|---|---|---|
| **Linear Algebra Problems in Quantum Computation** | | **Grand Ballroom** |
| 10:00 - 10:30 | Edward Poon | On maximally entangled states |
| 10:30 - 11:00 | Seung-Hyeok Kye | Facial structures for bipartite separable states and applications to the separability criterion |
| 11:00 - 11:30 | Shmuel Friedland | On separable and entangled states in QIS |
| 11:30 - 12:00 | Xiaofei Qi | Constructing optimal entanglement witnesses by permutations |
| **Randomized Matrix Algorithms** | | **Bristol A** |
| 10:00 - 10:30 | Ilse Ipsen | Accuracy of Leverage Score Estimation |
| 10:30 - 11:00 | Michael Mahoney | Implementing Randomized Matrix Algorithms in Parallel and Distributed Environments |
| 11:00 - 11:30 | Haim Avron | A Randomized Asynchronous Linear Solver with Provable Convergence Rate |
| 11:30 - 12:00 | Alex Gittens | Randomized low-rank approximations, in theory and practice |
| **Linear Algebra, Control, and Optimization** | | **Bristol B** |
| 10:00 - 10:30 | Biswa Datta | A Hybrid Method for Time-Delayed Robust and Minimum Norm Quadratic Partial Eigenvalue Assignment |
| 10:30 - 11:00 | Paul Van Dooren | On finding the nearest stable system |
| 11:00 - 11:30 | Melvin Leok | Variational Integrators for Matrix Lie Groups with Applications to Geometric Control |
| 11:30 - 12:00 | Volker Mehrmann | Optimal control of descriptor systems |
| **Contributed Session on Computational Science** | | **Tiverton** |
| 10:00 - 10:20 | Youngmi Hur | A New Algebraic Approach to the Construction of Multidimensional Wavelet Filter Banks |
| 10:20 - 10:40 | Margarida Mitjana | Green matrices associated with Generalized Linear Polyominoes |
| 10:40 - 11:00 | Matthias Bolten | Multigrid methods for high-dimensional problems with tensor structure |
| 11:00 - 11:20 | Hashim Saber | A Model Reduction Algorithm for Simulating Sedimentation Velocity Analysis |
| 11:20 - 11:40 | Angeles Carmona | Discrete Serrin's Problem |
| 11:40 - 12:00 | Andrey Melnikov | A new method for solving completely integrable PDEs |
| **Matrix Methods for Polynomial Root-Finding** | | **Ocean** |
| 10:00 - 10:30 | David Watkins | Fast computation of eigenvalues of companion, comrade, and related matrices |
| 10:30 - 11:00 | Jared Aurentz | A Factorization of the Inverse of the Shifted Companion Matrix |
| 11:00 - 11:30 | Yuli Eidelman | Multiplication of matrices via quasiseparable generators and eigenvalue iterations for block triangular matrices |
| 11:30 - 12:00 | Matthias Humet | A generalized companion method to solve systems of polynomials |
| **Multilinear Algebra and Tensor Decompositions** | | **Patriots** |
| 10:00 - 10:30 | Charles Van Loan | Properties of the Higher-Order Generalized Singular Value Decomposition |
| 10:30 - 11:00 | Martin Mohlenkamp | Wrestling with Alternating Least Squares |
| 11:00 - 11:30 | Manda Winlaw | A Preconditioned Nonlinear Conjugate Gradient Algorithm for Canonical Tensor Decomposition |
| 11:30 - 12:00 | Eugene Tyrtyshnikov | Tensor decompositions and optimization problems |

## Tuesday, 01:20PM - 02:05PM

**Ivan Oseledets** **Location: Grand Ballroom**
**Numerical tensor methods and their applications**

TBA

## Tuesday, 02:15PM - 04:15PM

| | | |
|---|---|---|
| **Matrices and Graph Theory** | | **Grand Ballroom** |
| 2:15 - 2:45 | Minerva Catral | Zero Forcing Number and Maximum Nullity of Subdivided Graphs |
| 2:45 - 3:15 | Seth Meyer | Minimum rank for circulant graphs |
| 3:15 - 3:45 | Adam Berliner | Minimum rank, maximum nullity, and zero forcing number of simple digraphs |
| 3:45 - 4:15 | Michael Young | Generalized Propagation Time of Zero Forcing |
| **Linear Complementarity Problems and Beyond** | | **Bristol A** |
| 2:15 - 2:45 | Richard Cottle | Three Solution Properties of a Class of LCPs: Sparsity, Elusiveness, and Strongly Polynomial Computability |
| 2:45 - 3:15 | Ilan Adler | The central role of bimatrix games and LCPs with P-matrices in the computational analysis of Lemke-resolvable LCPs |
| 3:15 - 3:45 | Jinglai Shen | Uniform Lipschitz property of spline estimation of shape constrained functions |
| 3:45 - 4:15 | Youngdae Kim | A Pivotal Method for Affine Variational Inequalities (PATHAVI) |
| **Linear Algebra, Control, and Optimization** | | **Bristol B** |
| 2:15 - 2:45 | Michael Overton | Stability Optimization for Polynomials and Matrices |
| 2:45 - 3:15 | Shankar Bhattacharyya | Linear Systems: A Measurement Based Approach |
| 3:15 - 3:45 | Melina Freitag | Calculating the $H_\infty$-norm using the implicit determinant method. |
| 3:45 - 4:15 | Bart Vandereycken | Subspace methods for computing the pseudospectral abscissa and the stability radius |
| **Contributed Session on Numerical Linear Algebra Part 1** | | **Tiverton** |
| 2:15 - 2:35 | Marko Petkovic | Gauss-Jordan elimination method for computing outer inverses |
| 2:35 - 2:55 | Jesse Barlow | Block Gram-Schmidt Downdating |
| 2:55 - 3:15 | James Lambers | Explicit high-order time stepping based on componentwise application of asymptotic block Lanczos iteration |
| 3:15 - 3:35 | Froilán Dopico | Structured eigenvalue condition numbers for parameterized quasiseparable matrices |
| **Matrix Methods for Polynomial Root-Finding** | | **Ocean** |
| 2:15 - 2:45 | Victor Pan | Real and Complex Polynomial Root-finding by Means of Eigensolving |
| 2:45 - 3:15 | Peter Strobach | The Fitting of Convolutional Models for Complex Polynomial Root Refinement |
| 3:15 - 3:45 | Leonardo Robol | Solving secular and polynomial equations: a multiprecision algorithm |
| 3:45 - 4:15 | Raf Vandebril | Companion pencil vs. Companion matrix: Can we ameliorate the accuracy when computing roots? |
| **Multilinear Algebra and Tensor Decompositions** | | **Patriots** |
| 2:15 - 2:45 | Shmuel Friedland | The number of singular vector tuples and the uniqueness of best $(r_1, \ldots, r_d)$ approximation of tensors |
| 2:45 - 3:15 | Andre Uschmajew | On convergence of ALS and MBI for approximation in the TT format |
| 3:15 - 3:45 | Sergey Dolgov | Alternating minimal energy methods for linear systems in higher dimensions. Part I: SPD systems. |
| 3:45 - 4:15 | Sergey Dolgov | Alternating minimal energy methods for linear systems in higher dimensions. Part II: Faster algorithm and application to nonsymmetric systems |

## Tuesday, 04:45PM - 06:45PM

| | | |
|---|---|---|
| **Matrices and Graph Theory** | | **Grand Ballroom** |
| 4:45 - 5:15 | Woong Kook | Logarithmic Tree Numbers for Acyclic Complexes |
| 5:15 - 5:45 | Bryan Shader | The $\lambda$-$\tau$ problem for symmetric matrices with a given graph |
| 5:45 - 6:15 | Louis Deaett | The rank of a positive semidefinite matrix and cycles in its graph |
| 6:15 - 6:45 | Colin Garnett | Refined Inertias of Tree Sign Patterns |
| **Applications of Tropical Mathematics** | | **Bristol A** |
| 4:45 - 5:15 | Meisam Sharify | Computation of the eigenvalues of a matrix polynomial by means of tropical algebra |
| 5:15 - 5:45 | James Hook | Tropical eigenvalues |
| 5:45 - 6:15 | Andrea Marchesini | Tropical bounds for eigenvalues of matrices |
| 6:15 - 6:45 | Nikolai Krivulin | Extremal properties of tropical eigenvalues and solutions of tropical optimization problems |
| **Contributed Session on Numerical Linear Algebra Part 2** | | **Bristol B** |
| 4:45 - 5:05 | Clemente Cesarano | Generalized Chebyshev polynomials |
| 5:05 - 5:25 | Surya Prasath | On Moore-Penrose Inverse based Image Restoration |
| 5:25 - 5:45 | Na Li | CP decomposition of Partial symmetric tensors |
| 5:45 - 6:05 | Yusaku Yamamoto | An Algorithm for the Nonlinear Eigenvalue Problem based on the Contour Integral |
| 6:05 - 6:25 | Luis Verde-Star | Construction of all discrete orthogonal and $q$-orthogonal classical polynomial sequences using infinite matrices and Pincherle derivatives |
| **Matrix Methods for Polynomial Root-Finding** | | **Ocean** |
| 4:45 - 5:15 | Luca Gemignani | The Ehrlich-Aberth method for structured eigenvalue refinement |
| 5:15 - 5:45 | Gianna Del Corso | A condensed representation of generalized companion matrices |
| 5:45 - 6:15 | Olga Holtz | Generalized Hurwitz matrices and forbidden sectors of the complex plane |
| **Multilinear Algebra and Tensor Decompositions** | | **Patriots** |
| 4:45 - 5:15 | Vladimir Kazeev | From tensors to wavelets and back |
| 5:15 - 5:45 | Misha Kilmer | Tensor-based Regularization for Multienergy X-ray CT Reconstruction |
| 5:45 - 6:15 | Lek-Heng Lim | Multiplying higher-order tensors |
| 6:15 - 6:45 | Lieven De Lathauwer | Between linear and nonlinear: numerical computation of tensor decompositions |

## Wednesday, 08:00AM - 08:45AM

**Thomas Laffey** **Location: Grand Ballroom**
**Characterizing the spectra of nonnegative matrices**

Given a list of complex numbers $\sigma := (\lambda_1, \ldots\ldots, \lambda_n)$, the nonnegative inverse eigenvalue problem (NIEP) asks for necessary and sufficient conditions for $\sigma$ to be the list of eigenvalues of an entry-wise nonnegative matrix. If $\sigma$ has this property, we say that it is *realizable.*

We will discuss the current status of research on this problem. We will also discuss the related problem, first studied by Boyle and Handelman in their celebrated work, of the realizability of $\sigma$ with an arbitrary number of zeros added.

With a particular emphasis on the properties of polynomials and power series constructed from $\sigma$, we will present some new results, obtained jointly with R. Loewy and H. Šmigoc, on constructive approaches to the NIEP, and some related observations of F. Holland.

For $\sigma$ above, write $f(x) = \Pi_{k=1}^{n}(x - \lambda_k)$. For realizable $\sigma$, V. Monov has proved interesting results on the realizability of the list of roots of the derivative $f'(x)$ and raised some intriguing questions. We will report some progress on these obtained jointly with my student Anthony Cronin.

## Wednesday, 08:45AM - 09:30AM

**Gilbert Strang** **Location: Grand Ballroom**
**Functions of Difference Matrices**

The second difference matrix $K$ is tridiagonal with entries $1, -2, 1$ down the diagonals. We can approximate the heat equation $u_t = u_{xx}$ on an interval by $u_t = Ku/h^2$. The solution involves $E = \exp(Kt)$ and we take a closer look at this matrix and related matrices for the wave equation: $C = \cos(\sqrt{-K}t)$ and $S = \mathrm{sinc}(\sqrt{-K}t)$. Sinc was a surprise.

The eigenvectors of $K$, $E$, $C$, $S$ are sine vectors. So $E_{ij}$ becomes a combination of sines times sines, weighted by eigenvalues of $E$. Rewriting in terms of $\sin((i-j)kh)$ and $\sin((i+j)kh)$, this matrix $E$ has a shift–invariant part (Toeplitz matrix): shift the initial function and the solution shifts. There is also a Hankel part that shifts the other way! We hope to explain this.

A second step is to find a close approximation to these sums by integrals. With smooth periodic functions the accuracy is superexponential. The integrals produce Bessel functions in the finite difference solutions to the heat equation and the wave equation. There are applications to graphs and networks.

## Wednesday, 10:00AM - 12:00PM

| | | |
|---|---|---|
| **Matrices and Orthogonal Polynomials** | | **Bristol A** |
| 10:00 - 10:30 | Paul Terwilliger | Leonard pairs and the $q$-Tetrahedron algebra |
| 10:30 - 11:00 | Edward Hanson | The tail condition for Leonard pairs |
| 11:00 - 11:30 | Antonio Duran | Wronskian type determinants of orthogonal polynomials, Selberg type formulas and constant term identities |
| 11:30 - 12:00 | Holger Dette | Optimal designs, orthogonal polynomials and random matrices |
| **Symbolic Matrix Algorithms** | | **Bristol B** |
| 10:00 - 10:30 | Gilles Villard | Euclidean lattice basis reduction: algorithms and experiments for disclosing integer relations |
| 10:30 - 11:00 | Arne Storjohann | Computing the invariant structure of integer matrices: fast algorithms into practice |
| 11:00 - 11:30 | Wayne Eberly | Early Termination over Small Fields: A Consideration of the Block Case |
| 11:30 - 12:00 | David Saunders | Wanted: reconditioned preconditioners |
| **Contributed Session on Numerical Range and Spectral Problems** | | **Tiverton** |
| 10:00 - 10:20 | Christopher Bailey | An exact method for computing the minimal polynomial |
| 10:20 - 10:40 | Yuri Nesterenko | Matrix canonical forms under unitary similarity transformations. Geometric approach. |
| 10:40 - 11:00 | Michael Karow | Inclusion theorems for pseudospectra of block triangular matrices |
| 11:00 - 11:20 | Xuhua Liu | Connectedness, Hessians and Generalized Numerical Ranges |
| **Matrices and Graph Theory** | | **Ocean** |
| 10:00 - 10:30 | Pauline van den Driessche | A Simplified Principal Minor Assignment Problem |
| 10:30 - 11:00 | Oliver Knill | The Dirac operator of a finite simple graph |
| 11:00 - 11:30 | Steve Kirkland | The Minimum Coefficient of Ergodicity for a Markov Chain with a Specified Directed Graph |
| 11:30 - 12:00 | Jason Molitierno | The Algebraic Connectivity of Planar Graphs |
| **Structure and Randomization in Matrix Computations** | | **Patriots** |
| 10:00 - 10:30 | Victor Pan | Transformations of Matrix Structures Work Again |
| 10:30 - 11:00 | Yousef Saad | Multilevel low-rank approximation preconditioners |
| 11:00 - 11:30 | Paul Van Dooren | On solving indefinite least squares-type problems via anti-triangular factorization |
| 11:30 - 12:00 | Francoise Tisseur | Structured matrix polynomials and their sign characteristic |

## Thursday, 08:00AM - 08:45AM

| **Fuzhen Zhang** | **Location: Grand Ballroom** |
|---|---|
| **Generalized Matrix Functions** | |

A generalized matrix function is a mapping from the matrix space to a number field associated with a subgroup of the permutation group and an irreducible character of the subgroup. Determinants and permanents are special cases of generalized matrix functions. Starting with a brief survey of certain elegant existing theorems, this talk will present some new ideas and results in the area.

## Thursday, 08:45AM - 09:30AM

| **Dianne O'Leary** | **Location: Grand Ballroom** |
|---|---|
| **Eckart-Young Meets Bayes: Optimal Regularized Low Rank Inverse Approximation** | |

We consider the problem of finding approximate inverses of a given matrix. We show that a regularized Frobenius norm inverse approximation problem arises naturally in a Bayes risk framework, and we derive the solution to this problem. We then focus on low-rank regularized approximate inverses and obtain an inverse regularized Eckart-Young-like theorem. Numerical experiments validate our results and show what can be gained over the use of sub-optimal approximations such as truncated SVD matrices.

## Thursday, 10:00AM - 12:00PM

| | | |
|---|---|---|
| **Matrices and Orthogonal Polynomials** | | **Grand Ballroom** |
| 10:00 - 10:30 | Luc Vinet | Harmonic oscillators and multivariate Krawtchouk polynomials |
| 10:30 - 11:00 | Jeff Geronimo | Matrix Orthogonal Polynomials in Bivariate Orthogonal Polynomials. |
| 11:00 - 11:30 | Manuel Manas | Scalar and Matrix Orthogonal Laurent Polynomials in the Unit Circle and Toda Type Integrable Systems |
| 11:30 - 12:00 | Hugo Woerdeman | Norm-constrained determinantal representations of polynomials |
| **Contributed Session on Numerical Linear Algebra Part 3** | | **Bristol A** |
| 10:00 - 10:20 | Kim Batselier | A Geometrical Approach to Finding Multivariate Approximate LCMs and GCDs |
| 10:20 - 10:40 | Xuzhou Chen | The Stationary Iterations Revisited |
| 10:40 - 11:00 | William Morrow | Matrix-Free Methods for Verifying Constrained Positive Definiteness and Computing Directions of Negative Curvature |
| 11:00 - 11:20 | Pedro Freitas | Derivatives of Functions of Matrices |
| 11:20 - 11:40 | Abderrahman Bouhamidi | Global Krylov subspace methods for computing the meshless elastic polyharmonic splines |
| 11:40 - 12:00 | Roel Van Beeumen | A practical rational Krylov algorithm for solving large-scale nonlinear eigenvalue problems |
| **Symbolic Matrix Algorithms** | | **Bristol B** |
| 10:00 - 10:30 | Victor Pan | Polynomial Evaluation and Interpolation: Fast and Stable Approximate Solution |
| 10:30 - 11:00 | Brice Boyer | Reducing memory consumption in Strassen-like matrix multiplication |
| 11:00 - 11:30 | Erich Kaltofen | Outlier detection by error correcting decoding and structured linear algebra methods |
| 11:30 - 12:00 | George Labahn | Applications of fast nullspace computation for polynomial matrices |
| **Contributed Session on Matrix Pencils** | | **Tiverton** |
| 10:00 - 10:20 | Javier Perez | New bounds for roots of polynomials from Fiedler companion matrices |
| 10:20 - 10:40 | Maria Isabel Bueno Cachadina | Symmetric Fiedler Pencils with Repetition as Strong Linearizations for Symmetric Matrix Polynomials. |
| 10:40 - 11:00 | Michal Wojtylak | Perturbations of singular hermitian pencils. |
| 11:00 - 11:20 | Andrii Dmytryshyn | Orbit closure hierarchies of skew-symmetric matrix pencils |
| 11:20 - 11:40 | Vasilije Perovic | Linearizations of Matrix Polynomials in Bernstein Basis |
| 11:40 - 12:00 | Susana Furtado | Skew-symmetric Strong Linearizations for Skew-symmetric Matrix Polynomials obtained from Fiedler Pencils with Repetition |
| **Matrices and Graph Theory** | | **Ocean** |
| 10:00 - 10:30 | Steven Osborne | Using the weighted normalized Laplacian to construct cospectral unweighted bipartite graphs |
| 10:30 - 11:00 | Steve Butler | Equitable partitions and the normalized Laplacian matrix |
| 11:00 - 11:30 | T. S. Michael | Matrix Ranks in Graph Theory |
| 11:30 - 12:00 | Shaun Fallat | On the minimum number of distinct eigenvalues of a graph |
| **Structure and Randomization in Matrix Computations** | | **Patriots** |
| 10:00 - 10:30 | Yuli Eidelman | The bisection method and condition numbers for quasiseparable of order one matrices |
| 10:30 - 11:00 | Luca Gemignani | The unitary eigenvalue problem |
| 11:00 - 11:30 | Marc Baboulin | Accelerating linear system solutions using randomization |
| 11:30 - 12:00 | Thomas Mach | Inverse eigenvalue problems linked to rational Arnoldi, and rational (non)symmetric Lanczos |

# Thursday, 01:30PM - 03:30PM

| | | |
|---|---|---|
| **Matrices and Orthogonal Polynomials** | | **Grand Ballroom** |
| 1:30 - 2:00 | Luis Velazquez | A quantum approach to Khrushchev's formula |
| 2:00 - 2:30 | Francisco Marcellan | CMV matrices and spectral transformations of measures |
| 2:30 - 3:00 | Marko Huhtanen | Biradial orthogonal polynomials and Complex Jacobi matrices |
| 3:00 - 3:30 | Rostyslav Kozhan | Spectral theory of exponentially decaying perturbations of periodic Jacobi matrices |
| **Linear and Nonlinear Perron-Frobenius Theory** | | **Bristol A** |
| 1:30 - 2:00 | Roger Nussbaum | Generalizing the Krein-Rutman Theorem |
| 2:00 - 2:30 | Helena Šmigoc | Constructing new spectra of nonnegative matrices from known spectra of nonnegative matrices |
| 2:30 - 3:00 | Julio Moro | The compensation game and the real nonnegative inverse eigenvalue problem |
| 3:00 - 3:30 | Yongdo Lim | Hamiltonian actions on the cone of positive definite matrices |
| **Linear Algebra Education Issues** | | **Bristol B** |
| 1:30 - 2:00 | Gilbert Strang | The Fundamental Theorem of Linear Algebra |
| 2:00 - 2:30 | Judi McDonald | The Role of Technology in Linear Algebra Education |
| 2:30 - 3:00 | Sandra Kingan | Linear algebra applications to data sciences |
| 3:00 - 3:30 | Jeffrey Stuart | Twenty years after The LACSG report, What happens in our textbooks and in our classrooms? |
| **Contributed Session on Nonnegative Matrices Part 1** | | **Tiverton** |
| 1:30 - 1:50 | Keith Hooper | An Iterative Method for detecting Semi-positive Matrices |
| 1:50 - 2:10 | Akiko Fukuda | An extension of the dqds algorithm for totally nonnegative matrices and the Newton shift |
| 2:10 - 2:30 | Anthony Cronin | The nonnegative inverse eigenvalue problem |
| 2:30 - 2:50 | Shani Jose | On Inverse-Positivity of Sub-direct Sums of Matrices |
| 2:50 - 3:10 | Prashant Batra | About negative zeros of entire functions, totally non-negative matrices and positive definite quadratic forms |
| 3:10 - 3:30 | Pietro Paparella | Matrix roots of positive and eventually positive matrices. |
| **Contributed Session on Graph Theory** | | **Ocean** |
| 1:30 - 1:50 | Caitlin Phifer | The Cycle Intersection Matrix and Applications to Planar Graphs |
| 1:50 - 2:10 | Milica Andelic | A characterization of spectrum of a graph and of its vertex deleted subgraphs |
| 2:10 - 2:30 | David Jacobs | Locating eigenvalues of graphs |
| 2:30 - 2:50 | Nathan Warnberg | Positive Semidefinite Propagation Time |
| 2:50 - 3:10 | Andres Encinas | Perturbations of Discrete Elliptic operators |
| 3:10 - 3:30 | Joseph Fehribach | Matrices and their Kirchhoff Graphs |
| **Structure and Randomization in Matrix Computations** | | **Patriots** |
| 1:30 - 2:00 | Jianlin Xia | Randomized and matrix-free structured sparse direct solvers |
| 2:00 - 2:30 | Ilse Ipsen | Randomly sampling from orthonormal matrices: Coherence and Leverage Scores |
| 2:30 - 3:00 | Victor Pan | On the Power of Multiplication by Random Matrices |
| 3:00 - 3:30 | Michael Mahoney | Decoupling randomness and vector space structure leads to high-quality randomized matrix algorithms |

## Thursday, 05:30PM - 07:30PM

| | | |
|---|---|---|
| **Linear and Nonlinear Perron-Frobenius Theory** | | **Grand Ballroom** |
| 5:30 - 6:00 | Marianne Akian | Policy iteration algorithm for zero-sum two player stochastic games: complexity bounds involving nonlinear spectral radii |
| 6:00 - 6:30 | Assaf Goldberger | Infimum over certain conjugated operator norms as a generalization of the Perron–Frobenius eigenvalue |
| 6:30 - 7:00 | Brian Lins | Denjoy-Wolff Type Theorems on Cones |
| 7:00 - 7:30 | Olga Kushel | Generalized Perron-Frobenius property and positivity of matrix spectra |
| **Matrix Inequalities** | | **Bristol A** |
| 5:30 - 6:00 | Fuzhen Zhang | Some Inequalities involving Majorization |
| 6:00 - 6:30 | Takashi Sano | Kwong matrices and related topics |
| 6:30 - 7:00 | Charles Johnson | Monomial Inequalities for p-Newton Matrices and Beyond |
| 7:00 - 7:30 | Tomohiro Hayashi | An order-like relation induced by the Jensen inequality |
| **Structured Matrix Functions and their Applications (Dedicated to Leonia Lerer on the occasion of his 70th birthday)** | | **Bristol B** |
| 5:30 - 6:00 | Leiba Rodman | One sided invertibility, corona problems, and Toeplitz operators |
| 6:00 - 6:30 | Gilbert Strang | Toeplitz plus Hankel (and Alternating Hankel) |
| 6:30 - 7:00 | Marinus Kaashoek | An overview of Leonia Lerer's work |
| **Contributed Session on Matrix Completion Problems** | | **Tiverton** |
| 5:30 - 5:50 | James McTigue | Partial matrices whose completions all have the same rank |
| 5:50 - 6:10 | Gloria Cravo | Matrix Completion Problems |
| 6:10 - 6:30 | Zheng QU | Dobrushin ergodicity coefficient for Markov operators on cones, and beyond |
| 6:30 - 6:50 | Ryan Wasson | The Normal Defect of Some Classes of Matrices |
| 6:50 - 7:10 | Yue Liu | Ray Nonsingularity of Cycle Chain Matrices |
| **Abstract Interpolation and Linear Algebra** | | **Ocean** |
| 5:30 - 6:00 | Sanne ter Horst | Rational matrix solutions to the Leech equation: The Ball-Trent approach revisited |
| 6:00 - 6:30 | Christopher Beattie | Data-driven Interpolatory Model Reduction |
| 6:30 - 7:00 | Johan Karlsson | Uncertainty and Sensitivity of Analytic Interpolation Methods in Spectral Analysis |
| 7:00 - 7:30 | Dan Volok | Multiscale stochastic processes: prediction and covariance extension |
| **Sign Pattern Matrices** | | **Patriots** |
| 5:30 - 6:00 | Zhongshan Li | Sign Patterns with minimum rank 2 and upper bounds on minimum ranks |
| 6:00 - 6:30 | Judi McDonald | Properties of Skew-Symmetric Adjacency Matrices |
| 6:30 - 7:00 | Leslie Hogben | Sign patterns that require or allow generalizations of nonnegativity |
| 7:00 - 7:30 | Craig Erickson | Sign patterns that require eventual exponential nonnegativity |

## Friday, 08:00AM - 08:45AM

**Dan Spielman** **Location: Grand Ballroom**
**Nearly optimal algorithms for solving linear equations in SDD matrices.**

We now have algorithms that solve systems of linear equations in symmetric, diagonally dominant matrices with m non-zero entries in time essentially O(m log m). These algorithms require no assumptions on the locations or values of these non-zero entries.

I will explain how these algorithms work, focusing on the algorithms of Koutis, Miller and Peng and of Kelner, Orecchia, Sidford, and Zhu.

## Friday, 08:45AM - 09:30AM

**Jean Bernard Lasserre** **Location: Grand Ballroom**
**Tractable characterizations of nonnegativity on a closed set via Linear Matrix Inequalities**

*Tractable characterizations* of polynomials (and even semi-algebraic functions) which are nonnegative on a set, is a topic of independent interest in Mathematics but is also of primary importance in many important applications, and notably in global optimization.

We will review two kinds of *tractable* characterizations of polynomials which are nonnegative on a basic closed semi-algebraic set $K \subset R^n$. Remarkably, both characterizations are through *Linear Matrix Inequalities* and can be checked by solving a hierarchy of semidefinite programs or generalized eigenvalue problems.

The first type of characterization is when knowledge on $K$ is through its defining polynomials, i.e., $K = \{x : g_j(x) \geqslant 0,\ j = 1, \ldots, m\}$, in which case some powerful certificates of positivity can be stated in terms of some sums of squares (SOS)-weighted representation. For instance, in global optimization this allows to define a hierarchy fo semidefinite relaxations which yields a monotone sequence of *lower bounds* converging to the global optimum (and in fact, finite convergence is generic).

Another (dual) way of looking at nonnegativity is when knowledge on $K$ is through *moments* of a measure whose support is $K$. In this case, checking whether a polynomial is nonnegative on $K$ reduces to solving a sequence of *generalized eigenvalue* problems associated with a countable (nested) family of real symmetric matrices of increasing size. When applied in global optimization over $K$, this results in a monotone sequence of *upper bounds* converging to the global minimum, which complements the previous sequence of lower bounds. These two (dual) characterizations provide convex *inner* (resp. *outer*) approximations (by spectrahedra) of the convex cone of polynomials nonnegative on $K$.

## Friday, 10:00AM - 12:00PM

| | | |
|---|---|---|
| **Matrices and Orthogonal Polynomials** | | **Grand Ballroom** |
| 10:00 - 10:30 | Carl Jagels | Laurent orthogonal polynomials and a QR algorithm for pentadiagonal recursion matrices |
| 10:30 - 11:00 | Vladimir Druskin | Finite-difference Gaussian rules |
| 11:00 - 11:30 | Lothar Reichel | Rational Krylov methods and Gauss quadrature |
| 11:30 - 12:00 | Raf Vandebril | A general matrix framework to predict short recurrences linked to (extended) Krylov spaces |
| **Matrix Inequalities** | | **Bristol A** |
| 10:00 - 10:30 | Tin-Yau Tam | On Ky Fan's result on eigenvalues and real singular values of a matrix |
| 10:30 - 11:00 | Fuad Kittaneh | A numerical radius inequality involving the generalized Aluthge transform |
| 11:00 - 11:30 | Rajesh Pereira | Two Conjectured Inequalities motivated by Quantum Structures. |
| 11:30 - 12:00 | Ameur Seddik | Characterizations of some distinguished subclasses of normal operators by operator inequalities |
| **Linear Algebra Education Issues** | | **Bristol B** |
| 10:00 - 10:30 | David Strong | Good first day examples in linear algebra |
| 10:30 - 11:00 | Rachel Quinlan | A proof evaluation exercise in an introductory linear algebra course |
| 11:00 - 11:30 | Megan Wawro | Designing instruction that builds on student reasoning in linear algebra: An example from span and linear independence |
| 11:30 - 12:00 | Sepideh Stewart | Teaching Linear Algebra in three Worlds of Mathematical Thinking |
| **Contributed Session on Nonnegative Matrices Part 2** | | **Tiverton** |
| 10:00 - 10:20 | James Weaver | Nonnegative Eigenvalues and Total Orders |
| 10:20 - 10:40 | Hiroshi Kurata | Some monotonicity results for Euclidean distance matrices |
| 10:40 - 11:00 | Maguy Trefois | Binary factorizations of the matrix of all ones |
| 11:00 - 11:20 | Ravindra Bapat | Product distance matrix of a tree with matrix weights |
| 11:20 - 11:40 | Naomi Shaked-Monderer | On the CP-rank and the DJL Conjecture |
| **Abstract Interpolation and Linear Algebra** | | **Ocean** |
| 10:00 - 10:30 | Izchak Lewkowicz | "Wrong" side interpolation by positive real rational functions |
| 10:30 - 11:00 | Quanlei Fang | Potapov's fundamental matrix inequalities and interpolation problems |
| 11:00 - 11:30 | Daniel Alpay | Quaternionic inner product spaces and interpolation of Schur multipliers for the slice hyperholomorphic functions |
| 11:30 - 12:00 | Vladimir Bolotnikov | Interpolation by contractive multipliers from Hardy space to weighted Hardy space |
| **Sign Pattern Matrices** | | **Patriots** |
| 10:00 - 10:30 | Marina Arav | Sign Vectors and Duality in Rational Realization of the Minimum Rank |
| 10:30 - 11:00 | Wei Gao | Sign patterns with minimum rank 3 |
| 11:00 - 11:30 | Timothy Melvin | Using the Nilpotent-Jacobian Method Over Various Fields and Counterexamples to the Superpattern Conjecture. |

## Friday, 01:20PM - 02:05PM

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|---|---|
| **Ravi Kannan** | **Location: Grand Ballroom** |
| **Randomized Matrix Algorithms** | |

A natural approach to getting a low-rank approximation to a large matrix is to compute with a randomly sampled sub-matrix. Initial theorems gave error bounds if we use sampling probabilities proportional to squared length of the rows. More sophisticated sampling approaches with probabilities based on leverage scores or volumes of simplices spanned by $k-$tuples of rows have led to proofs of better error bounds.

Recently, sparse random subspace embeddings have provided elegant solutions to low-rank approximations as well as linear regression problems. These algorithms fit into a computational model referred to as the Streaming Model and its multi-pass versions. Also, known theorems assert that not much improvement is possible in these models.

But, more recently, a Cloud Computing model has been shown to be more powerful for these problems yielding newer algorithms. The talk will survey some of these developments.

## Friday, 02:15PM - 04:15PM

| | | |
|---|---|---|
| **Structured Matrix Functions and their Applications (Dedicated to Leonia Lerer on the occasion of his 70th birthday)** | | **Grand Ballroom** |
| 2:15 - 2:45 | Joseph Ball | Structured singular-value analysis and structured Stein inequalities |
| 2:45 - 3:15 | Alexander Sakhnovich | Discrete Dirac systems and structured matrices |
| 3:15 - 3:45 | Hermann Rabe | Norm asymptotics for a special class of Toeplitz generated matrices with perturbations |
| 3:45 - 4:15 | Harm Bart | Zero sums of idempotents and Banach algebras failing to be spectrally regular |
| **Matrices and Total Positivity** | | **Bristol A** |
| 2:15 - 2:45 | Charles Johnson | The Totally Positive (Nonnegative) Completion Problem and Other Recent Work |
| 2:45 - 3:15 | Gianna Del Corso | qd-type methods for totally nonnegative quasiseparable matrices |
| 3:15 - 3:45 | Olga Kushel | Generalized total positivity and eventual properties of matrices |
| 3:45 - 4:15 | Rafael Canto | Quasi-$LDU$ factorization of totally nonpositive matrices. |
| **Linear Algebra Education Issues** | | **Bristol B** |
| 2:15 - 2:45 | Robert Beezer | A Modern Online Linear Algebra Textbook |
| 2:45 - 3:15 | Tom Edgar | Flipping the Technology in Linear Algebra |
| 3:15 - 3:45 | Sang-Gu Lee | Interactive Mobile Linear Algebra with Sage |
| 3:45 - 4:15 | Jason Grout | The Sage Cell server and other technology in teaching |
| **Matrix Inequalities** | | **Ocean** |
| 2:15 - 2:45 | Natalia Bebiano | Tsallis entropies and matrix trace inequalities in Quantum Statistical Mechanics |
| 2:45 - 3:15 | Takeaki Yamazaki | On some matrix inequalities involving matrix geometric mean of several matrices |
| 3:15 - 3:45 | Minghua Lin | Fischer type determinantal inequalities for accretive-dissipative matrices |
| **Contributed Session on Positive Definite Matrices** | | **Patriots** |
| 2:15 - 2:35 | Bala Rajaratnam | Regularization of positive definite matrices: Connections between linear algebra, graph theory, and statistics |
| 2:35 - 2:55 | Apoorva Khare | Sparse positive definite matrices, graphs, and absolutely monotonic functions |
| 2:55 - 3:15 | Dominique Guillot | Preserving low rank positive semidefinite matrices |

## Friday, 04:45PM - 06:45PM

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|---|---|---|
| **Structured Matrix Functions and their Applications (Dedicated to Leonia Lerer on the occasion of his 70th birthday)** | | **Grand Ballroom** |
| 4:45 - 5:15 | Ilya Spitkovsky | On almost normal matrices |
| 5:15 - 5:45 | David Kimsey | Trace formulas for truncated block Toeplitz operators |
| 5:45 - 6:15 | Hugo Woerdeman | Determinantal representations of stable polynomials |
| **Matrices and Total Positivity** | | **Bristol A** |
| 4:45 - 5:15 | Jorge Delgado | Accurate and fast algorithms for some totally nonnegative matrices arising in CAGD |
| 5:15 - 5:45 | Alvaro Barreras | Computations with matrices with signed bidiagonal decomposition |
| 5:45 - 6:15 | José-Javier Martínez | More accurate polynomial least squares fitting by using the Bernstein basis |
| 6:15 - 6:45 | Stephane Launois | Deleting derivations algorithm and TNN matrices |
| **Matrix Methods in Computational Systems Biology and Medicine** | | **Bristol B** |
| 4:45 - 5:15 | Lee Altenberg | Two-Fold Irreducible Matrices: A New Combinatorial Class with Applications to Random Multiplicative Growth |
| 5:15 - 5:45 | Natasa Durdevac | Understanding real-world systems using random-walk-based approaches for analyzing networks |
| 5:45 - 6:15 | Marcus Weber | Improved molecular simulation strategies using matrix analysis |
| 6:15 - 6:45 | Amir Niknejad | Stochastic Dimension Reduction Via Matrix Factorization in Genomics and Proteomics |
| **Contributed Session on Matrix Equalities and Inequalities** | | **Tiverton** |
| 4:45 - 5:05 | Bas Lemmens | Geometry of Hilbert's and Thompson's metrics on cones |
| 5:05 - 5:25 | Dawid Janse van Rensburg | Rank One Perturbations of $H$-Positive Real Matrices |
| 5:25 - 5:45 | Hosoo Lee | Sylvester equation and some special applications |
| 5:45 - 6:05 | Jadranka Micic Hot | Refining some inequalities involving quasi–arithmetic means |
| **Linear Algebra Problems in Quantum Computation** | | **Ocean** |
| 4:45 - 5:15 | Man-Duen Choi | Who's Afraid of Quantum Computers? |
| 5:15 - 5:45 | Panayiotis Psarrakos | An envelope for the spectrum of a matrix |
| 5:45 - 6:15 | Raymond Nung-Sing Sze | A Linear Algebraic Approach in Constructing Quantum Error Correction Code |
| **Applications of Tropical Mathematics** | | **Patriots** |
| 4:45 - 5:15 | Marianne Johnson | Tropical matrix semigroups |
| 5:15 - 5:45 | Pascal Benchimol | Tropicalizing the simplex algorithm |
| 5:45 - 6:15 | Marie MacCaig | Integrality in max-algebra |
| 6:15 - 6:45 | Viorel Nitica | Generating sets of tropical hemispaces |

# Minisymposia

### Abstract Interpolation and Linear Algebra

Organizer(s): Joseph Ball, Vladimir Bolotnikov

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|---|---|---|
| Thursday | 05:30PM – 07:30PM | Ocean |
| Friday | 10:00AM – 12:00PM | Ocean |

### Advances in Combinatorial Matrix Theory and its Applications

Organizer(s): Carlos Fonseca, Geir Dahl

| | | |
|---|---|---|
| Monday | 10:00AM – 12:00PM | Bristol A |
| Monday | 02:15PM – 04:15PM | Bristol A |
| Monday | 04:45PM – 06:45PM | Bristol A |

### Applications of Tropical Mathematics

Organizer(s): James Hook, Marianne Johnson, Sergei Sergeev

| | | |
|---|---|---|
| Tuesday | 04:45PM – 06:45PM | Bristol A |
| Friday | 04:45PM – 06:45PM | Patriots |

### Generalized Inverses and Applications

Organizer(s): Minerva Catral, Nestor Thome, Wimin Wei

| | | |
|---|---|---|
| Monday | 10:00AM – 12:00PM | Bristol B |
| Monday | 02:15PM – 04:15PM | Bristol B |

### Krylov Subspace Methods for Linear Systems

Organizer(s): James Baglama, Eric de Sturler

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|---|---|---|
| Monday | 02:15PM – 04:15PM | Patriots |
| Monday | 04:45PM – 06:45PM | Patriots |

### Linear Algebra Education Issues

Organizer(s): Avi Berman, Sang-Gu Lee, Steven Leon

| | | |
|---|---|---|
| Thursday | 01:30PM – 03:30PM | Bristol B |
| Friday | 10:00AM – 12:00PM | Bristol B |
| Friday | 02:15PM – 04:15PM | Bristol B |

### Linear Algebra Problems in Quantum Computation

Organizer(s): Chi-Kwong Li, Yiu Tung Poon

| | | |
|---|---|---|
| Monday | 10:00AM – 12:00PM | Grand Ballroom |
| Monday | 02:15PM – 04:15PM | Grand Ballroom |
| Monday | 04:45PM – 06:45PM | Grand Ballroom |
| Tuesday | 10:00AM – 12:00PM | Grand Ballroom |
| Friday | 04:45PM – 06:45PM | Ocean |

### Linear Algebra, Control, and Optimization

Organizer(s): Biswa Datta

| | | |
|---|---|---|
| Tuesday | 10:00AM – 12:00PM | Bristol B |
| Tuesday | 02:15PM – 04:15PM | Bristol B |

### Linear and Nonlinear Perron-Frobenius Theory

Organizer(s): Thomas J. Laffey, Bas Lemmens, Raphael Loewy

| | | |
|---|---|---|
| Thursday | 01:30PM – 03:30PM | Bristol A |
| Thursday | 05:30PM – 07:30PM | Grand Ballroom |

### Linear Complementarity Problems and Beyond

Organizer(s): Sou-Cheng Choi

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|---|---|---|
| Tuesday | 02:15PM – 04:15PM | Bristol A |

### Linear Least Squares Methods: Algorithms, Analysis, and Applications

Organizer(s): Sanzheng Qiao, Huaian Diao

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|---|---|---|
| Monday | 10:00AM – 12:00PM | Patriots |

### Matrices and Graph Theory

Organizer(s): Louis Deaett, Leslie Hogben

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|---|---|---|
| Tuesday | 02:15PM – 04:15PM | Grand Ballroom |
| Tuesday | 04:45PM – 06:45PM | Grand Ballroom |
| Wednesday | 10:00AM – 12:00PM | Ocean |
| Thursday | 10:00AM – 12:00PM | Ocean |

### Matrices and Orthogonal Polynomials

Organizer(s): Jeff Geronimo, Francisco Marcellan, Lothar Reichel

| | | |
|---|---|---|
| Wednesday | 10:00AM – 12:00PM | Bristol A |
| Thursday | 10:00AM – 12:00PM | Grand Ballroom |
| Thursday | 01:30PM – 03:30PM | Grand Ballroom |
| Friday | 10:00AM – 12:00PM | Grand Ballroom |

**Matrices and Total Positivity**

Organizer(s): Jorge Delgado, Shaun Fallat, Juan Manuel Pena

| | | |
|---|---|---|
| Friday | 02:15PM – 04:15PM | Bristol A |
| Friday | 04:45PM – 06:45PM | Bristol A |

**Matrices over Idempotent Semirings**

Organizer(s): Martin Gavalec, Sergei Sergeev

| | | |
|---|---|---|
| Monday | 04:45PM – 06:45PM | Bristol B |

**Matrix Inequalities**

Organizer(s): Minghua Lin, Fuzhen Zhang

| | | |
|---|---|---|
| Thursday | 05:30PM – 07:30PM | Bristol A |
| Friday | 10:00AM – 12:00PM | Bristol A |
| Friday | 02:15PM – 04:15PM | Ocean |

**Matrix Methods for Polynomial Root-Finding**

Organizer(s): Dario Bini, Yuli Eidelman, Marc Van Barel, Pavel Zhlobich

| | | |
|---|---|---|
| Tuesday | 10:00AM – 12:00PM | Ocean |
| Tuesday | 02:15PM – 04:15PM | Ocean |
| Tuesday | 04:45PM – 06:45PM | Ocean |

**Matrix Methods in Computational Systems Biology and Medicine**

Organizer(s): Konstantin Fackeldey, Amir Niknejad, Marcus Weber

| | | |
|---|---|---|
| Friday | 04:45PM – 06:45PM | Bristol B |

**Multilinear Algebra and Tensor Decompositions**

Organizer(s): Lieven De Lathauwer, Eugene Tyrtyshnikov

| | | |
|---|---|---|
| Tuesday | 10:00AM – 12:00PM | Patriots |
| Tuesday | 02:15PM – 04:15PM | Patriots |
| Tuesday | 04:45PM – 06:45PM | Patriots |

**Nonlinear Eigenvalue Problems**

Organizer(s): Froilan M. Dopico, Volker Mehrmann, Francoise Tisseur

| | | |
|---|---|---|
| Monday | 10:00AM – 12:00PM | Ocean |
| Monday | 02:15PM – 04:15PM | Ocean |
| Monday | 04:45PM – 06:45PM | Ocean |

**Randomized Matrix Algorithms**

Organizer(s): Ravi Kannan, Michael Mahoney, Petros Drineas, Nathan Halko, Gunnar Martinsson, Joel Tropp

| | | |
|---|---|---|
| Tuesday | 10:00AM – 12:00PM | Bristol A |

**Sign Pattern Matrices**

Organizer(s): Frank Hall, Zhongshan (Jason) Li, Hein van der Holst

| | | |
|---|---|---|
| Thursday | 05:30PM – 07:30PM | Patriots |
| Friday | 10:00AM – 12:00PM | Patriots |

**Structure and Randomization in Matrix Computations**

Organizer(s): Victor Pan, Jianlin Xia

| | | |
|---|---|---|
| Wednesday | 10:00AM – 12:00PM | Patriots |
| Thursday | 10:00AM – 12:00PM | Patriots |
| Thursday | 01:30PM – 03:30PM | Patriots |

**Structured Matrix Functions and their Applications (Dedicated to Leonia Lerer on the occasion of his 70th birthday)**

Organizer(s): Rien Kaashoek, Hugo Woerdeman

| | | |
|---|---|---|
| Thursday | 05:30PM – 07:30PM | Bristol B |
| Friday | 02:15PM – 04:15PM | Grand Ballroom |
| Friday | 04:45PM – 06:45PM | Grand Ballroom |

**Symbolic Matrix Algorithms**

Organizer(s): Jean-Guillaume Dumas, Mark Giesbrecht

| | | |
|---|---|---|
| Wednesday | 10:00AM – 12:00PM | Bristol B |
| Thursday | 10:00AM – 12:00PM | Bristol B |

# Abstracts

## Abstract Interpolation and Linear Algebra

### Quaternionic inner product spaces and interpolation of Schur multipliers for the slice hyperholomorphic functions

Daniel Alpay, Vladimir Bolotnikov, Fabrizio Colombo, Irene Sabadini

In the present paper we present some aspects of Schur analysis in the setting of slice hyperholomorphic functions. We study various interpolation problems for Schur multipliers. To that purpose, we first present results on quaternionic inner product spaces. We show in particular that a uniformly positive subspace in a quaternionic Krein space is ortho-complemented.

### Data-driven Interpolatory Model Reduction

Christopher Beattie

Dynamical systems are a principal tool in the modeling and control of physical processes in engineering and the sciences. Due to increasing complexity of underlying mathematical models, model reduction has become an indispensable tool in harnessing simulation in the design/experiment refinement cycle, particularly those that involve parameter studies and uncertainty analysis. This presentation focusses on the development of efficient interpolatory model reduction strategies that are capable of producing optimal reduced order models. Particular attention is paid to Loewner matrix methods that utilize empirical data provided either by physical experiments or by ancillary simulations.

### Interpolation by contractive multipliers from Hardy space to weighted Hardy space

Vladimir Bolotnikov

We will show that any contractive multiplier from the Hardy space to a weighted Hardy space can be factored as a fixed factor composed with the classical Schur multiplier (contractive multiplier between Hardy spaces). The result will be then applied to get a realization result for weighted Hardy-inner functions and, on the other hand, some results on interpolation for a Hardy-to-weighted-Hardy contractive multiplier class. The talk is based on a joint work with Joseph A. Ball.

### Potapov's fundamental matrix inequalities and interpolation problems

Quanlei Fang

As an extension of Potapovs approach of fundamental matrix inequalities to interpolation problems, abstract interpolation problems have been studied extensively since 1980s. We will discuss some aspects of possible generalizations of the Potapov fundamental matrix inequalities and related interpolation problems.

### Uncertainty and Sensitivity of Analytic Interpolation Methods in Spectral Analysis

Johan Karlsson, Tryphon Georgiou

Spectral analysis of a time series is often posed as an analytic interpolation problem based on estimated second-order statistics. Here the family of power spectra which are consistent with a given range of interpolation values represents the uncertainty set about the "true" power spectrum. Our aim is to quantify the size of this uncertainty set using suitable notions of distance, and in particular, to compute the diameter of the set since this represents an upper bound on the distance between any choice of a nominal element in the set and the "true" power spectrum. Since the uncertainty set may contain power spectra with lines and discontinuities, it is natural to quantify distances in the weak topology—the topology defined by continuity of moments. We provide examples of such weakly-continuous metrics and focus on particular metrics for which we can explicitly quantify spectral uncertainty.

We then specifically consider certain high resolution techniques which utilize Nevanlinna-Pick interpolation, and compute worst-case a priori uncertainty bounds solely on the basis of the interpolation points. This allows for a priori selection of the interpolation points for minimal uncertainty over selected frequency bands.

### "Wrong" side interpolation by positive real rational functions

Daniel Alpay, Vladimir Bolotnikov, Izchak Lewkowicz

We here show that arbitrary $m$ nodes in the open left half of the complex plane, can always be mapped to anywhere in the complex plane by rational positive real functions of degree of at most, $m$. Moreover we introduce a parameterization of all these interpolating functions.

### Rational matrix solutions to the Leech equation: The Ball-Trent approach revisited

Sanne ter Horst

Using spectral factorization techniques, a method is given by which rational matrix solutions to the Leech equation with rational matrix data can be computed explicitly. This method is based on an approach by J.A. Ball and T.T. Trent, and generalizes techniques from recent work of T.T. Trent for the case of polynomial matrix data.

### Multiscale stochastic processes: prediction and covariance extension

Dan Volok

Prediction and covariance extension of multiscale stochastic processes, which arise in the wavelet filter theory, require an efficient algorithm for inversion of positive matrices with a certain structure. We shall discuss such an algorithm (a recursion of the Schur-Levinson type) and its applications. This talk is based on joint work with Daniel Alpay.

## Advances in Combinatorial Matrix Theory and its Applications

### Several types of Bessel numbers generalized and some combinatorial setting

Gi-Sang Cheon

The Bessel numbers and the Stirling numbers have a long history and both have been generalized in a variety of directions. Here we present a second level generalization that has both as special cases. This generalization often preserves the inverse relation between the first and second kind and has simple combinatorial interpretations. We also frame the discussion in terms of the exponential Riordan group and Riordan matrices. Then the inverse relation is just the matrix inverse and factoring inside the group leads to many results connecting the various Stirling and Bessel numbers.

This work is jointed with Ji-Hwan Jung and Louis Shapiro.

### On the P-sets of acyclic matrices

Zhibin Du, Carlos da Fonseca

In this talk we characterize those trees for which the number of P-vertices is maximal.

### Majorization transforms and Ryser's algorithm

Geir Dahl

The notion of a transfer is central in the theory of majorization; in particular it lies behind the characterization of majorization in terms of doubly stochastic matrices. We introduce a new form of transfer and prove some of its properties. Moreover, we discuss a connection to Ryser's algorithm for constructing (0,1)-matrices with given row and column sums.

### Eigenvalues, Multiplicities and Linear Trees

Charles Johnson

Let H(G) be the collection of all Hermitian matrices with graph G. When G is a tree, T, there are many important results about the collection of multiplicity lists for the eigenvalues among matrices in H(T). However, some even stronger statements begin to fail for rather large trees. This seems to be not because of the number of vertices, but because the stronger statements are true for "linear" trees. (All trees on fewer than 10 vertices are linear.) We mention some recent work on linear trees and their multiplicity lists.

### Patterns of Alternating Sign Matrices

Kathleen Kiernan, Richard Brualdi, Seth Meyer, Michael Schroeder

We look at some properties of the zero-nonzero patterns of alternating sign matrices (ASMs). In particular, we show the minimal term rank of an $n \times n$ ASM is $\lceil 2\sqrt{n+1} - 2 \rceil$. We also discuss maximal ASMs, with respect to the notion of ASM extension. An ASM $A$ is an ASM extension of ASM $B$ if the matrices are the same size, and any non-zero entries of $B$ are strictly contained within the non-zero entries of $A$.

### Creating a user-satisfaction index

In-Jae Kim, Brian Barthel

In this talk it is shown how to use PageRank centrality to construct a parsimonious survey instrument, and then how to create a user-satisfaction index based on the survey using multivariate methods.

### Good Bases for Nonnegative Realizations

Judi McDonald

Nonnegative and eventually nonnegative matrices are important in many applications. The essence of nonnegativity is two-fold it depends on the spectrum and the basis of eigenvectors. In this talk, good potential bases for nonnegative and eventually nonnegative matrices will be illustrated, as well as effective ways to organize these bases.

### Large set of equiangular lines in Euclidean spaces

Ferenc Szollosi

A set of lines, represented by the unit vectors $v_1, v_2, \ldots, v_k$ are called equiangular, if there is a constant $d$ such that $|\langle v_i, v_j \rangle| = d$ for all $1 \leqslant i < j \leqslant k$. In this talk we give an overview of the known constructions of maximal set of equiangular lines and present a new general lower bound on the number of equiangular lines in real Euclidean spaces.

### A combinatorial approach to nearly uncoupled Markov chains

Ryan Tifenbach

A Markov chain is a sequence of random variables $x_0, x_1, \ldots$ that take on values in a state space $\mathcal{S}$. A set $\mathcal{A} \subseteq \mathcal{S}$ is referred to as an almost invariant aggregate if transitions from $x_t$ to $x_{t+1}$ where $x_t \in \mathcal{A}$ and $x_{t+1} \notin \mathcal{A}$ are exceedingly rare. A Markov chain is referred to as nearly uncoupled if there are two or more disjoint almost invariant aggregates contained in its state space. Nearly uncoupled Markov chains are characterised by long periods of relatively constant behaviour, punctuated by sudden, extreme changes. We present algorithms for producing almost invariant aggregates of a nearly uncoupled Markov chain, given in the form of a stochastic matrix. These algorithms combine a combinatorial approach with the utilisation of the stochastic complement – a tool from the theory of stochastic matrices.

### Companion Matrices and Spectrally Arbitrary Patterns

Kevin Vander Meulen, Brydon Eastman, In-Jae Kim, Bryan Shader

We explore some sparse spectrally arbitrary matrix patterns. We focus on patterns with entries in $\{*, \#, 0\}$, with $*$ indicating that the entry is a nonzero real number and $\#$ representing an arbitrary real number. As such, the standard companion matrix of order $n$ can be considered to be a spectrally arbitrary $\{*, \#, 0\}$-pattern with $(n-1)$ $*$ entries and $n$ $\#$ entries. We characterize all patterns which, like the standard companion matrices, uniquely realize every possible spectrum for a real matrix. We compare some of our work to recent results on companion matrices (including Fiedler companion matrices), as well as existing research on spectrally arbitrary zero-nonzero patterns.

## Applications of Tropical Mathematics

### Tropicalizing the simplex algorithm

Xavier Allamigeon, Pascal Benchimol, Stephane Gaubert, Michael Joswig

We present an analogue of the simplex algorithm, allowing one to solve tropical (max-plus) linear programming problems. This algorithm computes, using signed tropical Cramer determinants, a sequence of feasible solutions which coincides with the image by the valuation of the sequence produced by the classical simplex algorithm over the ordered field of Puiseux series. This is motivated in particular by (deterministic) mean payoff games (which are equivalent to tropical linear programming).

### Tropical eigenvalues

James Hook, Francoise Tisseur

Tropical eigenvalues can be used to approximate the order of magnitude of the eigenvalues of a classical matrix or matrix polynomial. This approximation is useful when the classical eigenproblem is badly scaled. For example

$$A = \begin{bmatrix} 10^5 & -10^4 \\ i & 10^{-2} \end{bmatrix}, \quad \mathrm{Val}(A) = \begin{bmatrix} 5 & 4 \\ 0 & -2 \end{bmatrix}.$$

The classical matrix $A$ has eigenvalues $\lambda_1 = (-1 + 0.000001i) \times 10^5$ and $\lambda_2 = (-0.01 - i) \times 10^{-1}$. By taking the componentwise-log-of-absolute-value $A$ corresponds to the max-plus matrix $\mathrm{Val}(A)$, which has tropical eigenvalues $\mathrm{t}\lambda_1 = 5$ and $\mathrm{t}\lambda_2 = -1$.

In this talk I will give an overview of the theory of tropical eigenvalues for matrices and matrix polynomials emphasising the relationship between tropical and classical spectra.

### Tropical matrix semigroups

Marianne Johnson

In this talk I will discuss some of my recent research with Mark Kambites and Zur Izhakian. In particular, I will give some structural results about the semigroup of 'tropical matrices', that is, the set of all matrices with real entries, with respect to a 'tropical' multiplication defined by replacing the usual addition of numbers by maximisation, and replacing the usual multiplication of numbers by addition. I will give some information about the *idempotent* tropical matrices (that is, the matrices which square (tropically) to themselves) and their corresponding groups.

### Extremal properties of tropical eigenvalues and solutions of tropical optimization problems

Nikolai Krivulin

We consider linear operators on finite-dimensional semimodules over idempotent semifields (tropical linear operators) and examine extremal properties of their spectrum. It turns out that the minimum value for some nonlinear functionals that involve tropical linear operators and use multiplicative conjugate transposition is determined by their maximum eigenvalue, where the minimum and maximum are thought in the sense of the order induced by the idempotent addition in the carrier semifield. We exploit the extremal properties to solve multidimensional optimization problems formulated in the tropical mathematics setting as to minimize the functionals under constraints in the form of linear equations and inequalities. Complete closed-form solutions are given by using a compact vector representation. Applications of the results to real-world problems are presented, including solutions to both unconstrained and constrained multidimensional minimax single facility location problems with rectilinear and Chebyshev distances.

### Integrality in max-algebra

Peter Butkovic, Marie MacCaig

The equations $Ax \leqslant b$, $Ax = b$, $Ax \leqslant \lambda x$, $Ax = \lambda x$,

$Ax = By$ and $Ax = Bx$ have been extensively studied in max-algebra and methods to find solutions from $\overline{\mathbb{R}}$ are known. We consider the existence and description of integer solutions to these equations. First we show how existing methods for finding real solutions to these equations can be adapted to create methods for finding integer solutions or determining none exist. Further, in the cases when these methods do not provide a polynomial algorithm, we present a generic case which admits a strongly polynomial method for finding integer solutions.

### Tropical bounds for eigenvalues of matrices

Andrea Marchesini, Marianne Akian, Stephane Gaubert

We show that the product of the moduli of the k largest eigenvalues of a matrix is bounded from above by the product of its k largest tropical eigenvalues, up to a combinatorial constant which is expressed in terms of a permanental compound matrix. For k=1, this specializes to an inequality of Friedland (1986). We then discuss the optimality of such upper bounds for certain special classes of matrices, and give conditions under which analogous lower bounds can also be obtained.

### Generating sets of tropical hemispaces

Viorel Nitica, Sergey Sergeev, Ricardo Katz

In this paper we consider tropical hemispaces, defined as tropically convex sets whose complements are also tropically convex, and tropical semispaces, defined as maximal tropically convex sets not containing a given point. We characterize tropical hemispaces by means of generating sets, composed of points and rays, that we call (P;R)-representations. With each hemispace we associate a matrix with coefficients in the completed tropical semiring, satisfying an extended rank-one condition. Our proof techniques are based on homogenization (lifting a convex set to a cone), and the relation between tropical hemispaces and semispaces.

### Computation of the eigenvalues of a matrix polynomial by means of tropical algebra

Marianne Akian, Dario Bini, Stephane Gaubert, Vanni Noferini, Meisam Sharify

We show that the moduli of the eigenvalues of a matrix polynomial are bounded by the tropical roots. These tropical roots, which can be computed in linear time, are the non-differentiability points of an auxiliary tropical polynomial, or equivalently, the opposites of the slopes of its Newton polygon. This tropical polynomial depends only on the norms of the matrix coefficients. We extend to the case of matrix polynomials some bounds obtained by Hadamard, Ostrowski and Polya for the roots of scalar polynomials. These results show that the tropical roots can provide an a priori estimation of the moduli of the eigenvalues which can be applied in the numerical computation of the eigenvalues. We developed a scaling method by using tropical roots and we showed experimentally that this scaling improves the backward stability of the computations, particularly in situations in which the data have various orders of magnitude. We also showed that using tropical roots as the initial approximations in Ehrlich-Aberth method, can substantially decrease the number of iterations in the numerical computation of the eigenvalues.

## Generalized Inverses and Applications

### Generalized inverses and extensions of the Sherman-Morrison-Woodbury formulae

Nieves Castro-Gonzalez, M. Francisca Martinez-Serrano, Juan Robles

The Sherman-Morrison-Woodbury (SMW) formulae relate the inverse of a matrix after a small rank perturbation to the inverse of the original matrix. Various authors extended the SMW formulae for some cases when the matrix is singular or rectangular.

In this work we focus on complex matrices of the type $A + YGZ^*$ and we develop expressions for the Moore-Penrose inverse of this sum. We obtain generalizations of the formula

$$(A + YGZ^*)^\dagger = A^\dagger - A^\dagger Y (G^\dagger + Z^* A^\dagger Y)^\dagger Z^* A^\dagger,$$

which was established under certain conditions. We survey some known results on generalized inverses and their applications to the SMW-type formulae and present several extensions.

### Matrix groups constructed from $\{R, s{+}1, k\}$-potent matrices

Minnie Catral, Leila Lebtahi, Jeffrey Stuart, Nestor Thome

A matrix $A \in \mathbb{C}^{n\times n}$ is called $\{R, s+1, k\}$-potent if $A$ satisfies $RA = A^{s+1}R$, where $R \in \mathbb{C}^{n\times n}$ and $R^k = I_n$. Such matrices were introduced and studied in [L. Lebtahi, J. Stuart, N. Thome, J.R. Weaver. Matrices $A$ such that $RA = A^{s+1}R$ when $R^k = I$, *Linear Algebra and its Applications*, http://dx.doi.org/10.1016/j.laa.2012.10.034, in press, 2013], and one of the properties noted in that paper was that these matrices are generalized group invertible. The aim of this talk is to construct a matrix group arising from a fixed $\{R, s+1, k\}$-potent matrix and study some properties of this group.

### Reverse order law for the generalized inverses

Dragana Cvetkovic-Ilic

The reverse order law for the Moore Penrose inverse seems to have been studied first by Greville , in the '60s , giving a necessary and sufficient condition for the reverse order law

$$(AB)^\dagger = B^\dagger A^\dagger \qquad (1)$$

for matrices $A$ and $B$. This result was followed by many interesting other results concerning the reverse order law for the weighted Moore-Penrose inverse, the reverse order law for the product of three or more matrices or in general case the reverse order law for K-inverses, where $K \subseteq \{1,2,3,4\}$.

We shall examine the reverse order law for $\{1,3\}$, $\{1,4\}$, $\{1,2,3\}$ and $\{1,2,4\}$-inverses in the setting of $C^*$-algebras, and in that case we will present purely algebraic necessary and sufficient conditions under which this type of reverse order law holds. In the case of $\{1,3,4\}$–inverses, we will prove that $(AB)\{1,3,4\} \subseteq B\{1,3,4\} \cdot A\{1,3,4\} \Leftrightarrow (AB)\{1,3,4\} = B\{1,3,4\}\cdot A\{1,3,4\}$. Also, we will consider the mixed type reverse order law and the absorption laws for the generalized inverses.

### On invertibility of combinations of $k$-potent operators

Chunyuan Deng

In this talk, we will report some recent results on the general invertibility of the products and differences of projectors and generalized projectors. The invertibility, the group invertibility and the $k$-potency of the linear combinations of $k$-potents are investigated, under certain commutativity properties imposed on them. In addition, the range relations of projectors and the detailed representations for various inverses are presented.

### On the Level-2 Condition Number for MooreCPenrose Inverse in Hilbert Space

Huaian Diao, Yimin Wei

We prove that $cond_\dagger(T)-1 \leqslant cond_\dagger^{[2]}(T) \leqslant cond_\dagger(T)+1$ where $T$ is a linear operator in a Hilbert space, $cond_\dagger(T)$ is the condition number of computing its Moore-Penrose inverse, and $cond_\dagger^{[2]}(T)$ is the level-2 condition number of this problem.

### Formulas for the Drazin inverse of block anti-triangular matrices

Esther Dopazo, M.Francisca Martinez-Serrano, Juan Robles

Let $A$ be an $n \times n$ complex matrix. The Drazin inverse of $A$ is the unique matrix $A^D$ satisfying the relations:

$$A^DAA^D = A^D, \quad A^DA = AA^D, \quad A^{k+1}A^D = A,$$

where $k = ind(A)$ is the smallest nonnegative integer such that $rank(A^k) = rank(A^{k+1})$. If $ind(A) = 1$, then $A^D$ is the group inverse of $A$. The concept of Drazin inverse plays an important role in various fields like Markov chains, singular differential and difference equations, iterative methods, etc.

Deriving expressions for generalized inverses of matrices has been a topic that has received considerable attention in the literature over the last few years.

In particular, the problem of obtaining explicit formulas for the inverse of Drazin and the group inverse (in case of existence) of a $2 \times 2$ block matrix $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$, where $A$ and $D$ are square matrices, was posed as an open problem by Campbell and Meyer in 1979. Since then, an important formula has been given for the case of block triangular matrices, and recently some partial results have been obtained under specific conditions, but the general problem still remains open.

In 1983, Campbell proposed the problem of obtaining representations of the Drazin inverse of anti-triangular matrices of the form $M = \begin{pmatrix} A & B \\ C & 0 \end{pmatrix}$, in conecction with the problem to find general expressions for the solutions of second-order systems of the differential equations. Furthermore, these kind of matrices appear in other applications like graph theory, saddle-point problems and semi-iterative methods.

Therefore, in this work we focus on deriving formulas for the Drazin inverse of $2 \times 2$ anti-triangular matrices which extend results given in the literature. Moreover the proposed formulas will be applied to some special structured matrices.

This research has been partly supported by Project MTM2010-18057, “Ministerio de Ciencia e Innovación” of Spain.

### Representations for the generalized Drazin inverse of block matrices in Banach algebras

Dijana Mosic

We present explicit representations of the generalized Drazin inverse of a block matrix with the generalized Schur complement being generalized Drazin invertible in a Banach algebra under some conditions. The provided results extend earlier works given in the literature.

## Krylov Subspace Methods for Linear Systems

### Restarted Block Lanczos Bidiagonalization Algorithm for Finding Singular Triplets

James Baglama

A restarted block Lanczos bidiagonalization method is described for computing, a few of the largest, smallest, or near a specified positive number, singular triplets of a large rectangular matrix. Leja points are used as shifts in the implicitly restarted Lanczos bidiagonalization method. The new method is often computational faster than other implicitly restarted Lanczos bidiagonalization methods, requires a much smaller storage space, and can be used to find interior singular values. Computed examples show the new method to be competitive with available schemes.

### The convergence behavior of BiCG

Eric de Sturler, Marissa Renardy

The Bi-Conjugate Gradient method (BiCG) is a well-known Krylov method for linear systems of equations, proposed about 35 years ago. It forms the basis for some of the most successful iterative methods today, like BiCGSTAB. Nevertheless, the convergence behavior is poorly understood. The method satisfies a Petrov-Galerkin property, and hence its residual is constrained to a space of decreasing dimension. However, that does not explain why, for many problems, the method converges in, say, a hundred or a few hundred iterations for problems involving a hundred thousand or a million unknowns. For many problems, BiCG converges not much slower than an optimal method, like GMRES, even though the method does not satisfy any optimality properties. In fact, Anne Greenbaum showed that every three-term recurrence, for the first (n/2)+1 iterations (for a system of dimension n), is BiCG for some initial 'left' starting vector. So, why does the method work so well in most cases? We will introduce Krylov methods, discuss the convergence of optimal methods, describe the BiCG method, and provide an analysis of its convergence behavior.

### Robust and Scalable Schur Complement Method using Hierarchical Parallelism

Xiaoye Li, Ichitaro Yamazaki

Modern numerical simulations give rise to sparse linear equations that are becoming increasingly difficult to solve using standard techniques. Matrices that can be directly factorized are limited in size due to large memory and inter-node communication requirements. Preconditioned iterative solvers require less memory and communication, but often require an effective preconditioner, which is not readily available. The Schur complement based domain decomposition method has great potential for providing reliable solution of large-scale, highly-indefinite algebraic equations.

We developed a hybrid linear solver PDSLin, Parallel Domain decomposition Schur complement based LINear solver, which employs techniques from both sparse direct and iterative methods. In this method, the global system is first partitioned into smaller interior subdomain systems, which are connected only through the separators. To compute the solution of the global system, the unknowns associated with the interior subdomain systems are first eliminated to form the Schur complement system, which is defined only on the separators. Since most of the fill occurs in the Schur complement, the Schur complement system is solved using a preconditioned iterative method. Then, the solution on the subdomains is computed by using the partial solution on the separators and solving another set of subdomain systems.

For a scalable implementation, it is imperative to exploit multiple levels of parallelism; namely, solving independent subdomains in parallel and using multiple processors per subdomain. This hierarchical parallelism can maintain numerical stability as well as scalability. In this multilevel parallelization framework, performance bottlenecks due to load imbalance and excessive communication occur at two levels: in an intra-processor group assigned to the same subdomain and among inter-processor groups assigned to different subdomains. We developed several techniques to address these issues, such as taking advantage of the sparsity of right-hand-side columns during sparse triangular solutions with interfaces, load balancing sparse matrix-matrix multiplication to form update matrices, and designing an effective asynchronous point-to-point communication of the update matrices. We present experimental results to demonstrate that our hybrid solver can efficiently solve very large highly-indefinite linear systems on thousands of processors.

### GMRES convergence bounds that depend on the right-hand side vector

David Titley-Peloquin, Jennifer Pestana, Andrew Wathen

GMRES is one of the most popular Krylov subspace methods for solving linear systems of equations, $Bx = b$, $B \in \mathbb{C}^{n\times n}$, $b \in \mathbb{C}^n$. However, obtaining convergence bounds for GMRES that are generally descriptive is still considered a difficult problem. In this talk, we introduce bounds for GMRES applied to systems with nonsingular, diagonalizable coefficient matrices that explicitly include the initial residual vector. The first of these involves a polynomial least-squares problem on the spectrum of $B$, while the second reduces to an ideal GMRES problem on a rank-one modification of the diagonal matrix of eigenvalues of $B$. Numerical experiments demonstrate that these bounds can accurately describe GMRES convergence.

### Implicitly Restarting the LSQR Algorithm

James Baglama, Daniel Richmond

LSQR is an iterative algorithm for solving large sparse least squares problems $\min_x \|b - Ax\|_2$. For certain matrices $A$, LSQR can exhibit slow convergence. Results in this presentation pertain to a strategy used to speed up the convergence of LSQR, namely, using an implicitly restarted Golub-Kahan bidiagonalization. The restarting is used to improve the search space and is carried out by applying the largest harmonic Ritz values as implicit shifts while simultaneously using LSQR to compute the solution to the minimization problem. Theoretical results show why this strategy is advantageous and numerical examples show this method to be competitive with existing methods.

### Using ILU(0) to estimate the diagonal of the in-

**verse of a matrix.**

Andreas Stathopoulos, Lingfei Wu, Jesse Laeuchli, Stratis Gallopoulos, Vasilis Kalatzis

For very large, sparse matrices, the calculation of the diagonal or even the trace of their inverse is a very computationally intensive task that involves the use of Monte Carlo (MC). Hundreds or thousands of quadratures ($x^T A^{-1} x$) are averaged to obtain 1-2 digits of accuracy in the trace. An ILU(k) preconditioner can speed up the computation of the quadratures, but it can also be used to reduce the MC variance (e.g., by estimating the diagonal of $A^{-1} - (LU)^{-1}$). Such variance reduction may not be sufficient, however. We take a different approach based two observations.

First, there is an old, but not much used algorithm to obtain the diagonal of $M = (LU)^{-1}$ when $L, U$ are sparse, in time linear to the number of nonzero elements of $L, U$. For some matrices, the factors from the ILU(0) preconditioner provide a surprisingly accurate estimate for the trace of $A^{-1}$. Second, we observed that, in general, the diag($M$) captures the pattern (if not the individual values) of the diag($A^{-1}$) very well. Therefore, if we solved for the exact $A^{-1}_{jj} = e_j^T A^{-1} e_j$, for $j = 1, \ldots, k$ anchor points, we could find an interpolating function $f(M_{jj}, j) \approx A^{-1}_{jj}$, for all $j$. Preliminary experiments have shown that $k = 100$ is often enough to achieve $O(10^{-2})$ accuracy in the trace estimation, much faster than classical MC.

### MPGMRES: a generalized minimum residual method with multiple preconditioners

Chen Greif, Tyrone Rees, Daniel Szyld

Standard Krylov subspace methods only allow the user to choose a single preconditioner, although in many situations there may be a number of possibilities. Here we describe an extension of GMRES, multi-preconditioned GMRES, which allows the use of more than one preconditioner. We give some theoretical results, propose a practical algorithm, and present numerical results from problems in domain decomposition and PDE-constrained optimization. These numerical experiments illustrate the applicability and potential of the multi-preconditioned approach.

## Linear Algebra Education Issues

### A Modern Online Linear Algebra Textbook

Robert Beezer

*A First Course in Linear Algebra* is an open source textbook which debuted in 2004. The use of an open license makes it possible to construct, distribute and employ the book in ways that would be impossible with a proprietary text.

I will briefly describe the organization of the book and some novel features of the content. Several major technological enhancements were made to the book in the last six months of 2012, which have resulted in an online version that is unlike any other mathematics textbook. I will describe and demonstrate these changes, how they were implemented and discuss how they have changed the way I teach introductory linear algebra.

### Flipping the Technology in Linear Algebra

Tom Edgar

There are many excellent textbooks that incorporate the use of Sage or other computer algebra packages for a Linear Algebra course. What if you prefer another text, are part of a multi-section course, or your department does not give you the freedom to choose your textbook? In this talk we will discuss the possibilities and practicalities of incorporating the use of technology (in particular Sage) in any linear algebra class. We detail our experience of creating mini-videos to teach the basics of Sage and designing out-of-class projects that require computer use to enhance students understanding of key concepts or to introduce them to applications of linear algebra. Using the “flipped classroom” idea for technology provides the possibility of using technology in any class without changing the structure of the class. In particular, we will describe the process of creating videos, discuss the problems and successes we encountered, and make suggestions for future classes. Though we have only limited experience with this method, we also intend to describe some of the initial, perceived pedagogical benefits to incorporating technology into courses in this way.

### The Sage Cell server and other technology in teaching

Jason Grout

We will discuss uses of technology in teaching linear algebra, including use of the new Sage Cell Server (`https://sagecell.sagemath.org`). Sage (`http://sagemath.org`) is a free open-source math software system that includes both numerical and theoretical linear algebra capabilities. For example, in addition to standard matrix and vector operations over various base rings and fields, you can also directly create and manipulate vector spaces and linear transformations. The Sage Cell server makes it easy to embed arbitrary Sage or Octave (Matlab-like) computations and interactive explorations into a web page, as links in a pdf file, or as QR codes in a printed book. Using technology can be a powerful way to allow student exploration and to reinforce classroom discussion.

### Linear algebra applications to data sciences

Sandra Kingan

Typical data sciences courses list elementary statistics and linear algebra as prerequisites, but the linear algebra that students are expected to know includes, for example, multiple regression, singular value decomposition, principal component analysis, and latent semantic indexing. These topics are rarely covered in a first linear algebra course. As a result, for many students these techniques become a few lines of code accompanied by a vague understanding. In this talk I will describe the development of a rigorous course on the mathematical methods for analyzing data that also incorporates modern real-world examples. Development of this course was supported by an NSF TUES grant.

### Interactive Mobile Linear Algebra with Sage

Sang-Gu Lee

For over 20 years, the issue of using an adequate computer algebra system (CAS) tool in teaching and learning of linear algebra has been raised constantly. A variety of CAS tools were introduced in several linear algebra textbooks. However, due to some realistic problems in Korea, it has not been introduced in the class and the theoretical aspect of linear algebra has been focused in teaching and learning of it. We have tried to find or make a simple and comprehensive method for using ICT in our classes.

Nowadays, most of our students have a smartphone. We found that Sage could be an excellent solution for them to use it for mathematical computation and simulation. To achieve our objectives, we started to develop web contents and smartphone apps to design our lectures in mobile environment since 2009. We have developed tools for Mobile mathematics with smartphones in teaching of linear algebra. Furthermore, we offered several QR codes to our students with the goal of helping the student to learn some linear algebra concepts via problems solving.

In this talk, we introduce our mobile infrastructure of the Sage and how ONE could use it in linear algebra classes. Our talk will focus on what we have done in our linear algebra class with interactive mobile-learning environment.

### The Role of Technology in Linear Algebra Education

Judi McDonald

What are the opportunities and challenges created by modern technology? What role should technology play in Linear Algebra Education? How/when is it advantageous to use a computer for presentations? When are hand written "live" presentations better? What should go into e-books? How do professors handle the fact that students can "google" the solutions? What are the advantages and disadvantages of online homework? How can clickers be used effectively in the classroom? As you can see, I have many questions about where modern technology fits into the learning of linear algebra and will use this talk to get faculty thinking about both the opportunities and the challenges

### A proof evaluation exercise in an introductory linear algebra course

Rachel Quinlan

In a first year university linear algebra course, students were presented with five purported proofs of the statement that every linear transformation of $R^2$ fixes the origin. The students were familiar with the definition of a linear transformation as an additive function that respects scalar multiplication, and with the matrix representation of a linear transformation of $R^2$. They were asked to consider each argument and decide on its correctness. They were also asked to rank the five "proofs" in order of preference and to comment on their rankings.

Some recent work of K. Pfeiffer proposes a theoretical frame for the consideration of proof evaluation and of students' responses to tasks of this kind. With this in mind, we discuss our students' assessments of the five presented arguments. It will be suggested that exercises of this nature can give us some useful understanding of our students' thinking about the mechanisms and purposes of proof and (in this instance) about some essential concepts of linear algebra.

### Teaching Linear Algebra in three Worlds of Mathematical Thinking

Sepideh Stewart

In my PhD thesis I created and applied a theoretical framework combining the strengths of two major mathematics education theories (Talls three worlds and APOS) in order to investigate the learning of linear algebra concepts at the university level. I produced and analyzed a large data set concerning students embodied and symbolic ways of thinking about elementary linear algebra concepts. It was anticipated that this study may provide suggestions with the potential for widespread positive consequences for learning mathematics. In this talk I highlight aspects of this research as well as my current project on employing Talls theory together with Schoenfelds theory of Resources, Orientations and Goals (ROGs) to address the teaching aspects of linear algebra.

### The Fundamental Theorem of Linear Algebra

Gilbert Strang

This theorem is about the four subspaces associated with an m by n matrix A of rank r.

**Part 1** (Row rank = Column rank) The column spaces of $A$ and $A^T$ have the same dimension $r$. I would like to give my two favorite proofs (admitting that one came from Wikipedia).

**Part 2** The nullspaces of $A^T$ and $A$ are orthogonal to those column spaces, with dimensions $m-r$ and $n-r$.

**Part 3** The SVD gives orthonormal bases for the four subspaces, and in those bases $A$ and $A^T$ are diagonal:

$Av_j = \sigma_j u_j$ and $A^T u_j = \sigma_j v_j$ with $\sigma_j > 0$ for $1 \leqslant j \leqslant r$. This step involves the properties of symmetric matrices $A^T A$ and $AA^T$. Maybe it is true that understanding this theorem is the goal of a linear algebra course. The best example of the SVD is a first difference matrix $A$.

### Good first day examples in linear algebra

David Strong

In any mathematics class, examples that involve multiple concepts are of great educational value. Such an example is of even more use if it is simple enough to be introduced on the first day of class. I will discuss a few good examples that I have used in my own first semester linear algebra course.

### Twenty years after The LACSG report, What happens in our textbooks and in our classrooms?

Jeffrey Stuart

In 1993, David Carlson, Charles Johnson, David Lay and Duane Porter published the Linear Algebra Curriculum Study Group Recommendations for the First Course in Linear Algebra in the College Mathematics Journal. Their paper laid out what ought to be emphasized (or de-emphasized) in a first course, and consequently, in the textbook for a first course. An examination of the multitude of linear algebra books currently on the market (at least in the USA) shows that at least some of the LACSG recommendations have come to pass: all recent texts that this author has examined emphasize $\mathbb{R}^n$ and matrices over abstract vector spaces and linear functions. Matrix multiplication is usually discussed in terms of linear combinations of columns. The needs of client disciplines are often reflected in a variety of applied problems, and in some texts, entire sections or chapters. Numerical algorithms abound. And yet anyone who has taught from one of the over-stuffed calculus textbooks knows that what is in a textbook can differ greatly from what is covered or emphasized in a course. We will review the content of linear algebra textbooks in the USA, and propose a survey of mathematics departments to better understand what is actually being taught in linear algebra courses.

### Designing instruction that builds on student reasoning in linear algebra: An example from span and linear independence

Megan Wawro

Research into student thinking in mathematics is often used to inform curriculum design and instructional practices. I articulate such an example in the form of an instructional sequence that supports students' reinvention of the concepts of span and linear independence. The sequence, the creation of which was guided by the instructional design theory of Realistic Mathematics Education, builds on students informal and intuitive ways of reasoning towards more conventional, generalizable, and abstract ways of reasoning. During the presentation I will share the instructional sequence and samples of student work, as well as discuss underlying theoretical influences that guided this work.

## Linear Algebra Problems in Quantum Computation

### Rank reduction for the local consistency problem

Jianxin Chen

We address the problem of how simple a solution can be for a given quantum local consistency instance. More specifically, we investigate how small the rank of the global density operator can be if the local constraints are known to be compatible. We prove that any compatible local density operators can be satisfied by a low rank global density operator. Then we study both fermionic and bosonic versions of the N-representability problem as applications. After applying the channel-state duality, we prove that any compatible local channels can be obtained through a global quantum channel with small Kraus rank. This is joint work with Zhengfeng Ji, Alexander Klyachko, David W. Kribs, and Bei Zeng.

### Who's Afraid of Quantum Computers?

Man-Duen Choi

Suddenly, there arrives the new era of quantum computers, with the most frightening effects of quantum optics. In this expository talk, I will present my own experience of a pure mathematicians adventure in quantum wonderland. In particular, I seek sense and sensibility of quantum entanglements, with pride and prejudice in matrix theory.

### On separable and entangled states in QIS

Shmuel Friedland

In this talk we discuss the notion of separable and entangled states in quantum information science. First we discuss the decidability problem: find if the state is separable or not. Second we discuss how to find the distance, called the relative entropy, from an entangled state to the set of separable states. We will concentrate on the bipartite case.

### Quantum measurements and maps preserving strict convex combinations and pure states

Jinchuan Hou, Lihua Yang

Let $\mathcal{S}(H)$ be the convex set of all states (i.e., the positive operators with trace one) on a complex Hilbert space $H$. It is shown that a map $\psi : \mathcal{S}(H) \to \mathcal{S}(K)$ with

$2 \leqslant \dim H < \infty$ preserves pure states and strict convex combinations if and only if $\psi$ has one of the forms: (1) $\rho \mapsto \sigma_0$ for any $\rho \in \mathcal{S}(H)$; (2) $\psi(\mathcal{P}ur(H)) = \{Q_1, Q_2\}$; (3) $\rho \mapsto \frac{M\rho M^*}{\mathrm{Tr}(M\rho M^*)}$, where $\sigma_0$ is a pure state on $K$ and $M : H \to K$ is an injective linear or conjugate linear operator. For multipartite systems, we also give a structure theorem for maps that preserve separable pure states and strict convex combinations. These results allow us to characterize injective (local) quantum measurements and answer some conjectures proposed in [J.Phys.A:Math.Theor.45 (2012) 205305].

**Linear preserver problems arising in quantum information science**

Ajda Fosner, Zejun Huang, Chi-Kwong Li, Yiu-Tung Poon, Nung-Sing Sze

For a positive integer $n$, let $M_n$ and $H_n$ be the set of $n \times n$ complex matrices and Hermitian matrices, respectively. Suppose $m, n \geqslant 2$ are positive integers. In this talk, we will characterize the linear maps $\phi$ on $M_{mn}$ ($H_{mn}$) such that

$$f(\phi(A \otimes B)) = f(A \otimes B)$$

for all $A \in M_m$ ($H_m$) and $B \in M_n$ ($H_n$), where $f(X)$ is the spectrum, the spectral radius, the numerical radius, a higher numerical range, a Ky Fan norm or a Schatten norm of $X$.

**On the Minimum Size of Unextendible Product Bases**

Nathaniel Johnston, Jianxin Chen

An unextendible product basis is a set of mutually orthogonal product vectors in a tensor product Hilbert space such that there is no other product vector orthogonal to them all. A long-standing open question asks for the minimum number of vectors needed to form an unextendible product basis given the local Hilbert space dimensions. A partial solution was found by Alon and Lovasz in 2001, but since then only a few other cases have been solved. We solve this problem in many special cases, including the case where the Hilbert space is bipartite (i.e., it is the tensor product of just two local Hilbert spaces), as well as the qubit case (i.e., where every local Hilbert space is 2-dimensional).

**Private quantum subsystems and connections with quantum error correction**

David Kribs

No abstract entered.

**Facial structures for bipartite separable states and applications to the separability criterion**

Seung-Hyeok Kye

One of the most important criterion for separability is given by the operation of partial transpose which is the one sided transpose for the tensor product of matrices. The PPT (positive partial transpose) criterion tells us that the partial transpose of a separable state must by positive semi-definite. In the convex set of all PPT states, the boundary between separability and entanglement consists of faces for separable states. This is why we are interested in the facial structures for the convex set of all separable states. The whole structures are far from being understood completely. In this talk, we will restrict ourselves to the two qutrit and qubit-qudit cases to see how the facial structures can be used to distinguish separable states from entangled states.

**Some non-linear preservers on quantum structures**

Lajos Molnar

In this talk we present some of our results mostly on non-linear transformations of certain quantum structures that can be viewed as different kinds of symmetries.

We discuss order isomorphisms on observables, spectral-order isomorphisms and sequential endomorphisms of the space of Hilbert space effects, transformations on quantum states which preserve the relative entropy, or a general quantum $f$-divergence.

**Transformations on density matrices preserving the Holevo bound**

Gergo Nagy, Lajos Molnar

Given a probability distribution $\{\lambda_1, \ldots, \lambda_m\}$, for any collection
$\{\rho_1, \ldots, \rho_m\}$ of density matrices the corresponding Holevo bound is defined by

$$\chi(\rho_1, \ldots, \rho_m) = S\left(\sum_{k=1}^{m} \lambda_k \rho_k\right) - \sum_{k=1}^{m} \lambda_k S(\rho_k),$$

where $S(\rho) = -\mathrm{Tr}\, \rho \log \rho$ is the von Neumann entropy. This is one of the most important numerical quantities appearing in quantum information science. In this talk we present the structure of all such maps $\phi$ on the set (or on any dense subset) of density matrices which leave the Holevo bound invariant i.e. which satisfy

$$\chi(\phi(\rho_1), \ldots, \phi(\rho_m)) = \chi(\rho_1, \ldots, \rho_m)$$

for all possible collections $\{\rho_1, \ldots, \rho_m\}$. It turns out that every such map is induced by a unitary matrix and hence our result is closely related to Wigner's famous theorem on the structure of quantum mechanical symmetry transformations. Another result of similar spirit involving quantum Tsallis entropy will also be presented.

**Decomposing a Unitary Matrix into a Product of Weighted Quantum Gates**

Chi-Kwong Li, Diane Christine Pelejo, Rebecca Roberts

Let $d = 2^n$ for some $n$. It is known that any unitary matrix $U \in M_d(\mathbb{C})$ can be written as a product of $d(d-1)/2$ fully controlled quantum gates (or two-level gates or Givens rotations). These gates are computationally costly and we wish to replace these expensive gates with more efficient ones. For $k = 0, 1, 2, \ldots, n-1$, we assign the cost or weight $k$ to a quantum gate with $k$ controls. We develop an algorithm to decompose any $d$-by-$d$ matrix into a combination of these gates with potentially minimal cost. We write a program that will implement this algorithm. Possible modifications to the problem will also be presented.

### On trumping

David Kribs, Rajesh Pereira, Sarah Plosker

Trumping is a partial order on vectors in $\mathbb{R}^n$ that generalizes the more familiar concept of majorization. Both of these partial orders have recently been considered in quantum information theory and shown to play an important role in entanglement theory. We explore connections between trumping and power majorization, a generalization of majorization that has been studied in the theory of inequalities. We also discuss extending the idea of trumping to continuous functions on $\mathbb{R}^n$, analogous to the notion of continuous majorization.

### On maximally entangled states

Edward Poon

A mixed state is said to be maximally entangled if it is a convex combination of maximally entangled pure states. We investigate the structure of maximally entangled states on a bipartite system, and of the space spanned by such states. We then apply our results to the problem of finding the linear preservers of such states.

### An envelope for the spectrum of a matrix

Panayiotis Psarrakos, Michael Tsatsomeros

We introduce and study an envelope-type region $\mathcal{E}(A)$ in the complex plane that contains the eigenvalues of a given $n \times n$ complex matrix $A$. $\mathcal{E}(A)$ is the intersection of an infinite number of regions defined by cubic curves. The notion and method of construction of $\mathcal{E}(A)$ extend the notion of the numerical range of $A$, $F(A)$, which is known to be an intersection of an infinite number of half-planes; as a consequence, $\mathcal{E}(A)$ is contained in $F(A)$ and represents an improvement in localizing the spectrum of $A$.

### Constructing optimal entanglement witnesses by permutations

Xiaofei Qi, Jinchuan Hou

A linear map $\Phi_D : M_n \to M_n$ is called a $D$-type map if $\Phi_D$ has the form $(a_{ij}) \longmapsto \mathrm{diag}(f_1, f_2, ..., f_n) - (a_{ij})$ with $(f_1, f_2, ..., f_n) = (a_{11}, a_{22}, ..., a_{nn})D$ for an $n \times n$ nonnegative matrix $D = (d_{ij})$ (i.e., $d_{ij} \geqslant 0$ for all $i, j$). For any permutation $\pi$ of $\{1, 2, \ldots, n\}$ and $t \geqslant 0$, let $\Phi_{t,\pi} : M_n \to M_n$ be the $D$-type map of the form with $D = (n-t)I_n + tP_\pi$. In this talk, we give necessary and sufficient conditions for $\Phi_{t,\pi}$ becoming positive, indecomposable and optimal, respectively.

### A Linear Algebraic Approach in Constructing Quantum Error Correction Code

Raymond Nung-Sing Sze

In the study of quantum error correction, quantum information scientists are interested in constructing practical and efficient schemes for overcoming the decoherence in quantum systems. In particular, researchers would like to obtain useful quantum correction codes and construct simple encoding and decoding quantum circuits. These problems can be reformulated and studied in a linear algebraic approach. In this talk, we review some recent work in quantum error correction along this direction.

This talk is based on joint work with Chi-Kwong Li, Mikio Nakahara, Yiu-Tung Poon, and Hiroyuki Tomita.

### Gradient Flows for the Minimum Distance to the Sum of Adjoint Orbits

Xuhua Liu, Tin-Yau Tam

Let $G$ be a connected semisimple Lie group and $\mathfrak{g}$ its Lie algebra. Let $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ be the Cartan decomposition corresponding to a Cartan involution $\theta$ of $\mathfrak{g}$. The Killing form $B$ induces a positive definite symmetric bilinear form $B_\theta$ on $\mathfrak{g}$ defined by $B_\theta(X, Y) = -B(X, \theta Y)$. Given $A_0, A_1, \ldots, A_N \in \mathfrak{g}$, we consider the optimization problem

$$\min_{k_i \in K} \left\| \sum_{i=1}^{N} \mathrm{Ad}\,(k_i)A_i - A_0 \right\|,$$

where the norm $\|\cdot\|$ is induced by $B_\theta$ and $K$ is the connected subgroup of $G$ with Lie algebra $\mathfrak{k}$. We obtain the gradient flow of the corresponding smooth function on the manifold $K \times \cdots \times K$. Our results give unified extensions of several results of Li, Poon, and Schulte-Herbrüggen. They are also true for reductive Lie groups.

### Higher dimensional witnesses for entanglement in $2N$–qubit chains

Justyna Zwolak, Dariusz Chruscinski

Entanglement is considered to be the most nonclassical manifestation of quantum physics and lies at the center of interest for physicists in the 21st century, as it is fundamental to future quantum technologies. Characterization of entanglement (i.e., delineating entangled from separable states) is equivalent to the characterization of positive, but not completely positive (PnCP), maps over matrix algebras.

For quantum states living in $2 \times 2$ and $2 \times 3$ dimensional spaces there exists a complete characterization of the separability problem (due to the celebrated Peres-Horodecki criterion). For increasingly higher dimensions, however, this task becomes increasingly difficult. There is considerable effort devoted to constructing PnCP maps, but a general procedure is still not known.

In our research, we are developing PnCP maps for higher dimensional systems. For instance, we recently generalized the Robertson map in a way that naturally meshes with $2N$ qubit systems, i.e., its structure respects the $2^{2N}$ growth of the state space. Using linear algebra techniques, we proved that this map is positive, but not completely positive, and also indecomposable and optimal. As such, it can be used to create witnesses that detect (bipartite) entanglement. We also determined the relation of our maps to entanglement breaking channels. As a byproduct, we provided a new example of a family of PPT (Positive Partial Transpose) entangled states. We will discuss this map as well as the new classes of entanglement witnesses.

## Linear Algebra, Control, and Optimization

### Linear Systems: A Measurement Based Approach

Lee Keel, Shankar Bhattacharyya

We present recent results obtained by us on the analysis, synthesis and design of systems described by linear equations. As is well known linear equations arise in most branches of science and engineering as well as social, biological and economic systems. The novelty of our approach is that no models of the system are assumed to be available, nor are they required. Instead, we show that a few measurements made on the system can be processed strategically to directly extract design values that meet specifications without constructing a model of the system, implicitly or explicitly. We illustrate these new concepts by applying them to linear DC and AC circuits, mechanical, civil and hydraulic systems, signal flow block diagrams and control systems. These applications are preliminary and suggest many open problems.

Our earlier research has shown that the representation of complex systems by high order models with many parameters may lead to fragility that is, the drastic change of system behaviour under infinitesimally small perturbations of these parameters. This led to research on model-free measurement based approaches to design. The results presented here are our latest effort in this direction and we hope they will lead to attractive alternatives to model-based design of engineering and other systems.

### A Hybrid Method for Time-Delayed Robust and Minimum Norm Quadratic Partial Eigenvalue Assignment

Biswa Datta

The quadratic partial eigenvalue assignment concerns reassigning a small set of eigenvalues of a large quadratic matrix pencil to suitably chosen ones by using feedback control, while leaving the remaining large number of eigenvalues and the associated eigenvectors unchanged. The problem arises in active vibration control of structures. For practical effectiveness, the feedback matrices have to be constructed in such a way that the both the feedback norms and the closed-loop eigenvalue sensitivity are minimized. These minimization problems give rise to difficult nonlinear optimization problems. Most of the existing solutions of these problems are based on the knowledge of system matrices, and do not make use of the "receptances" which are readily available from measurements. Furthermore, work on these eigenvalue assignment problems with time-delay in control input are rare.

In this paper, we propose a novel hybrid approach combining both system matrices and receptances to solve these problems both with and without-time delay. This hybrid approach is computationally more efficient than the approaches based on system matrices or the receptances alone. The numerical effectiveness of the proposed methods are demonstrated by results of numerical experiments.

### Calculating the $H_\infty$-norm using the implicit determinant method.

Melina Freitag

The $H_\infty$-norm of a transfer function is an important property for measuring robust stability. In this talk we present a new fast method for calculating this norm which uses the implicit determinant method. The algorithm is based on finding a two-dimensional Jordan block corresponding to a pure imaginary eigenvalue in a certain two-parameter Hamiltonian eigenvalue problem introduced by Byers (SIAM J. Sci. Statist. Comput., 9 (1988), pp. 875-881). At each step a structured linear system needs to be solved which can be done efficiently by transforming the problem into controller Hessenberg form. Numerical results show the performance of this algorithm for several examples and comparison is made with other methods for the same problem.

This is joint work with Paul Van Dooren (Louvain-la-Neuve) and Alastair Spence (Bath).

### Variational Integrators for Matrix Lie Groups with Applications to Geometric Control

Taeyoung Lee, Melvin Leok, N. Harris McClamroch

The geometric approach to mechanics serves as the theoretical underpinning of innovative control methodologies in geometric control theory. These techniques allow the attitude of satellites to be controlled using changes in its shape, as opposed to chemical propulsion, and are the basis for understanding the ability of a falling cat to always land on its feet, even when released in an inverted orientation.

We will discuss the application of geometric structure-preserving numerical schemes to the optimal control of mechanical systems. In particular, we consider Lie group variational integrators, which are based on a discretization of Hamilton's principle that preserves the Lie group structure of the configuration space. The importance of simultaneously preserving the symplectic and Lie group properties is also demonstrated.

**Optimal control of descriptor systems**

Peter Kunkel, Volker Mehrmann, Lena Scholz

We discuss the numerical solution of linear and nonlinear optimal control problems in descriptor form. We derive the necessary optimality conditions and discuss their solvability. The resulting optimality systems have a self-adjoint structure that can be used to identify the underlying symplectic flow and to compute the optimal solution in a structure preserving way.

**Stability Optimization for Polynomials and Matrices**

Michael Overton

Suppose that the coefficients of a monic polynomial or entries of a square matrix depend affinely on parameters, and consider the problem of minimizing the root radius (maximum of the moduli of the roots) or root abscissa (maximum of their real parts) in the polynomial case and the spectral radius or spectral abscissa in the matrix case. These functions are not convex and they are typically not locally Lipschitz near minimizers. We first address polynomials, for which some remarkable analytical results are available in one special case, and then consider the more general case of matrices, focusing on the static output feedback problem arising in control of linear dynamical systems. The polynomial results are joint with V. Blondel, M. Gurbuzbalaban and A. Megretski.

**On finding the nearest stable system**

Francois-Xavier Orban de Xivry, Yurii Nesterov, Paul Van Dooren

The nearest stable matrix to an unstable one is a difficult problem independently of the norm used. This is due to the nonconvex nature of the set of stable matrices as described by the Lyapunov equation and to the fact that we have to constrain simultaneously all the eigenvalues in the left-hand side of the complex plane. This talk presents a gradient based method to solve a reformulation of this problem for the Frobenius norm. The objective function features a smoothing term which enables us to obtain a convergence analysis which shows that a local minimum of the proposed reformulation is attained.

**Subspace methods for computing the pseudospectral abscissa and the stability radius**

Daniel Kressner, Bart Vandereycken

The pseudospectral abscissa and the stability radius are well-established tools for quantifying the stability of a matrix under unstructured perturbations. Based on first-order eigenvalue expansions, Guglielmi and Overton [SIAM J. Matrix Anal. Appl., 32 (2011), pp. 1166-1192] recently proposed a linearly converging iterative method for computing the pseudospectral abscissa. In this talk, we propose to combine this method and its variants with subspace acceleration. Each extraction step computes the pseudospectral abscissa of a small rectangular matrix pencil, which is comparably cheap and guarantees monotonicity. We prove local superlinear convergence for the resulting subspace methods. Moreover, these methods extend naturally to computing the stability radius. A number of numerical experiments demonstrate the robustness and efficiency of the subspace methods.

## Linear and Nonlinear Perron-Frobenius Theory

**Policy iteration algorithm for zero-sum two player stochastic games: complexity bounds involving nonlinear spectral radii**

Marianne Akian, Stephane Gaubert

Recent results of Ye and Hansen, Miltersen and Zwick show that policy iteration for one or two player zero-sum stochastic games, restricted to instances with a fixed and uniform discount rate, is strongly polynomial. We extend these results to stochastic games in which the discount rate can vanish, or even be negative. The restriction is now that the spectral radii of the nonnegative matrices associated to all strategies are bounded from above by a fixed constant strictly less than 1. This is the case in particular if the discount rates are nonnegative and if the life time for any strategy is bounded a priori. We also obtain an extension to a class of ergodic (mean-payoff) problems with irreducible stochastic transition matrices. Our results are based on methods of nonlinear Perron-Frobenius theory.

**Infimum over certain conjugated operator norms as a generalization of the Perron–Frobenius eigenvalue**

Assaf Goldberger

Let $||\cdot||$ denote the operator norm for a Euclidean space. If $A$ is a square matrix, we study the quantity $\mu_G(A) := \inf_{D\in G} ||DAD^{-1}||$ where $G$ is a subgroup of matrices. We mainly focus on two cases: Where $G$ is a subgroup of diagonal matrices, or where $G$ is a compact subgroup. In both cases we are able to show that $\mu_G(A)$ is a solution to a nonlinear eigenvalue problem on $A$. The value $\mu_G(A)$ satisfies few properties similar to the Perron–Frobenius value for nonnegative $A$, such as inheritence, Collatz-Wielandt bounds, and more. Part of this work is based on joint work with Miki Neumann.

**Generalized Perron-Frobenius property and positivity of matrix spectra**

Olga Kushel

We study a generalization of the Perron-Frobenius property with respect to a proper cone in $\mathbb{R}^n$. We examine matrices which possess the generalized Perron-frobenius property together with their compound matrices. We give examples of such matrices and apply the obtained theory to some matrix classes defined by determinantal inequalities.

**Hamiltonian actions on the cone of positive definite matrices**

Yongdo Lim

The semigroup of Hamiltonians acting on the cone of positive definite matrices via linear fractional transformations satisfies the Birkhoff contraction formula for the Thompson metric. In this talk we describe the action of the Hamiltonians lying in the boundary of the semigroup. This involves in particular a construction of linear transformations leaving invariant the cone of positive definite matrices (strictly positive linear mappings) parameterized over all square matrices. Its invertibility and relation to the Lyapunov and Stein operators are investigated in detail.

**Denjoy-Wolff Type Theorems on Cones**

Brian Lins

The classical Denjoy-Wolff theorem states that if an analytic self-map of the open unit disk in $\mathbf{C}$ has no fixed point, then all iterates of that map converge to a single point on the boundary of the unit disk. A direct generalization of this theorem applies to normalized iterates of order-preserving homogeneous of degree one maps $f : \operatorname{int} C \to \operatorname{int} C$ when $C$ is a strictly convex closed cone with nonempty interior in $\mathbf{R}^n$. If the map $f$ has no eigenvector in $\operatorname{int} C$, then all normalized iterates of $f$ converge to a single point on the boundary of $C$. A further generalization of this result applies when $f$ is defined on polyhedral cones. In that case, all limit points of the normalized iterates are contained in a convex subset of the boundary of $C$. We will survey some of the recent progress attempting to extend these Denjoy-Wolff type theorems to maps on other cones, particularly symmetric cones.

**The compensation game and the real nonnegative inverse eigenvalue problem**

Alberto Borobia, Julio Moro, Ricardo Soto

A connection is drawn between the problem of characterizing all possible real spectra of entrywise nonnegative matrices (the so-called real nonnegative inverse eigenvalue problem) and a game of combinatorial nature, related with compensation of negativity. The game is defined in such a a way that, given a set $\Lambda$ of real numbers, if the game with input $\Lambda$ has a solution then $\Lambda$ is realizable, i.e., it is the spectrum of some entrywise nonnegative matrix. This defines a special kind of realizability, which we call C-realizability. After observing that the set of all C-realizable sets is a strict subset of the set of realizable ones, we show that it strictly includes, in particular, all sets satisfying several previously known sufficient realizability conditions in the literature. Furthermore, the proofs of these conditions become much simpler when approached from this new point of view.

**Generalizing the Krein-Rutman Theorem**

Roger Nussbaum

The original Krein-Rutman theorem considers a closed, total cone in a real Banach space X and a continuous, compact linear map f of X to X such f(C) is contained in C. If r¿0, where r denotes the spectral radius of f, the Krein-Rutman theorem asserts that f has an eigenvector in C with eigenvalue r. For applications in a variety of directions (e.g., game theory, machine scheduling and mathematical biology)it is desirable to consider a closed cone C in a real Banach space and a map f of C to C which is continuous and homogeneous of degree one and preserves the partial ordering induced by C. To extend the Krein-Rutman theorem to this context, it is necessary to find appropriate definitions of the "cone spectral radius of f", r(f;C), and the "cone essential spectral radius of f", (rho)(f;C). We discuss what seem appropriate definitions of r(f;C) and (rho)(f;C) and mention some theorems and open questions concerning these quantities. If (rho)(f;C) is strictly less than r(f;C), we conjecture that f has an eigenvector in C with eigenvalue r(f;C). We describe some theorems which support this conjecture and mention some connections to an old,open conjecture in asymptotic fixed point theory.

**Constructing new spectra of nonnegative matrices from known spectra of nonnegative matrices**

Helena Šmigoc, Richard Ellard

The question, which lists of complex numbers are the spectrum of some nonnegative matrix, is known as the nonnegative inverse eigenvalue problem (NIEP). The list of complex numbers that is the spectrum of a nonnegative is called realizable.

In the talk we will present some methods on how to construct new realizable spectra from known realizable spectra. In particular, let

$$\sigma = (\lambda_1, a + ib, a - ib, \lambda_4, \dots, \lambda_n)$$

be realizable with the Perron eigenvalue $\lambda_1$. We will show how to use $\sigma$ to construct some new realizable lists of the from $(\mu_1, \mu_2, \mu_3, \mu_4, \lambda_4, \dots, \lambda_n)$. In the second part of the talk we will start with a realizable list

$$\sigma = (\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n)$$

with the Perron eigenvalue $\lambda_1$ and a real eigenvalue $\lambda_2$, and we will construct realizable spectra of the from $(\mu_1, \mu_2, \dots, \mu_k, \lambda_3, \dots, \lambda_n)$.

## Linear Complementarity Problems and Beyond

### The central role of bimatrix games and LCPs with P-matrices in the computational analysis of Lemke-resolvable LCPs

Ilan Adler

A major effort in linear complementarity theory has been the study of the Lemke algorithm. Over the years, many classes of linear complementarity problems (LCPs) were proven to be resolvable by the Lemke algorithm. For the majority of these classes it has been shown that these problems belong to the complexity class Polynomial Parity Arguments on Directed graphs (PPAD) and thus can be theoretically reduced to bimatrix games which are known to be PPAD-complete. In this talk we present a simple direct reduction of Lemke-resolvable LCPs to bimatrix games. Next, we present a reduction via perturbation of several major classes of PPAD LCPs, which are not known to be PPAD-complete, to LCPs with P-matrices. We conclude by discussing a major open problem in LCP theory regarding the question of how hard it is to solve LCP problems with P-matrices.

### Three Solution Properties of a Class of LCPs: Sparsity, Elusiveness, and Strongly Polynomial Computability

Ilan Adler, Richard Cottle, Jong-Shi Pang

We identify a class of Linear Complementarity Problems (LCPs) that are solvable in strongly polynomial time by Lemke's Algorithm (Scheme 1) or by the Parametric Principal Pivoting Method (PPPM). This algorithmic feature for the class of problems under consideration here is attributable to the proper selection of the covering vectors in Scheme 1 or the parametric direction vectors in the PPPM which lead to solutions of limited and monotonically nondecreasing support size, hence sparsity. These and other LCPs may very well have multiple solutions, many of which are unattainable by either algorithm and thus are said to be elusive.

### A Pivotal Method for Affine Variational Inequalities (PATHAVI)

Youngdae Kim, Michael Ferris,

PATHAVI is a solver for affine variational inequalities (AVIs) defined over a polyhedral convex set. It computes a solution by finding a zero of the normal map associated with a given AVI, which in this case is a piecewise linear equation. We introduce a path-following algorithm for solving the piecewise linear equation of the AVI using a pivotal method. The piecewise linear equation uses a starting point (an initial basis) computed using the Phase I procedure of the simplex method from any linear programming package. The main code follows a piecewise linear manifold in a complementary pivoting manner similar to Lemke's method. Implementation issues including the removal of the lineality space of a polyhedral convex set using LU factorization, repairing singular bases, and proximal point strategies will be discussed. Some experimental results comparing PATHAVI to other codes based on reformulations as a complementarity problem will be given.

### Uniform Lipschitz property of spline estimation of shape constrained functions

Jinglai Shen

In this talk, we discuss uniform properties of structured complementarity problems arising from shape restricted estimation and asymptotic analysis. Specifically, we consider B-spline estimation of a convex function in nonparametric statistics. The optimal spline coefficients of this estimator give rise to a family of size varying complementarity problems and Lipschitz piecewise linear functions. In order to show the uniform convergence of the estimator, we establish a critical uniform Lipschitz property in the infinity norm. Its implications in asymptotic statistical analysis of the convex B-spline estimator are revealed.

## Linear Least Squares Methods: Algorithms, Analysis, and Applications

### Structured Condition Numbers for a Linear Functional of the Tikhonov Regularized Solution

Huaian Diao, Yimin Wei

Both structured componentwise and normwise perturbation analysis of the Tikhonov regularization are presented. The structured matrices include: Toeplitz, Hankel, Vandermonde, and Cauchy matrices. Structured normwise, mixed and componentwise condition numbers for the Tikhonov regularization are introduced and exact expressions are derived. By means of the power method, the fast condition estimation algorithms are proposed. The structured condition numbers and perturbation bounds are tested on some numerical examples and compared with unstructured ones. The numerical examples illustrate the structured mixed condition numbers give sharper perturbation bounds than others.

### Inner-iteration preconditioning for the minimum-norm solution of rank-deficient linear systems

Keiichi Morikuni, Ken Hayami

We propose inner-iteration preconditioning for the right-preconditioned generalized minimal residual (AB-GMRES) method for solving the linear system of equations

$$A\boldsymbol{x} = \boldsymbol{b}, \tag{2}$$

where $A \in \mathbf{R}^{m\times n}$ may be rank-deficient: $\mathrm{rank}A < \min(m, n)$, and the system is consistent: $\boldsymbol{b} \in \mathcal{R}(A)$. Here, $\mathcal{R}(A)$ denotes the range of $A$. Preconditioners for Krylov subspace methods for (1) have been less studied in the rank-deficient case compared to the full-rank case. Here, we are particularly interested in obtaining the pseudo-inverse solution of (1), whose Euclidean norm is minimum, and which is useful in applications such as inverse problems and control.

We show that several steps of linear stationary iterative methods serve as inner-iteration preconditioners for AB-GMRES. Numerical experiments on large and sparse problems illustrate that the proposed method outperforms the preconditioned CGNE and LSMR methods in terms of the CPU time, especially in ill-conditioned and rank-deficient cases.

We also show that a sufficient condition for the proposed method to determine the pseudo-inverse solution of (1) without breakdown within $r$ iterations for arbitrary initial approximate solution $\boldsymbol{x}_0 \in \mathbf{R}^n$, where $r = \mathrm{rank}A$, is that the iteration matrix $H$ of the stationary inner iterations is semi-convergent, i.e., $\lim_{i\to\infty} H^i$ exists. The theory holds irrespective of whether $A$ is over- or under-determined, and whether $A$ is of full-rank or rank-deficient. The condition is satisfied by the Jacobi and successive over-relaxation (JOR and SOR) methods applied to the normal equations. Moreover, we use efficient implementations of these methods such as the NE-SOR method as inner iterations without explicitly forming the normal equations.

This is joint work with Ken Hayami of National Institute of Informatics and The Graduate University for Advanced Studies.

### Lattice Basis Reduction: Preprocessing the Integer Least Squares Problem

Sanzheng Qiao

The integer least squares (ILS) problem arises from numerous applications, such as geometry of numbers, cryptography, and wireless communications. It is known that the complexity of the algorithms for solving the ILS problem increases exponentially with the size of the coefficient matrix. To circumvent the high complexity problem, fast approximation methods have been proposed. However, the performance, measured by the error rate, of the approximation methods depends on the structure of the coefficient matrix, specifically, the condition number and/or the orthogonality defect. Lattice basis reduction, applied as a preprocessor on the coefficient matrix, can significantly improve the condition number and orthogonality defect, thus improve the performance. In this talk, after introducing lattices, bases, and basis reduction, we give a brief survey of the lattice basis reduction algorithms, then present our recent work.

### Stochastic conditioning of systems of equations

David Titley-Peloquin, Serge Gratton

Given nonsingular $A \in \mathbb{R}^{n\times n}$ and $b \in \mathbb{R}^n$, how sensitive is $x = A^{-1}b$ to perturbations in the data? This is a fundamental and well-studied question in numerical linear algebra. Bounds on $\|A^{-1}b - (A + E)^{-1}b\|$ can be stated using the condition number of the mapping $(A, b) \mapsto A^{-1}b$, provided $\|A^{-1}E\| < 1$. If $\|A^{-1}E\| \geqslant 1$ nothing can be said, as $A+E$ might be singular. These well-known results answer the question: how sensitive is $x$ to perturbations *in the worst case*? However, they say nothing of the *typical* or *average case* sensitivity.

We consider the sensitivity of $x$ to random noise $E$. Specifically, we are interested in properties of the random variable

$$\mu(A, b, E) = \|A^{-1}b - (A + E)^{-1}b\| = \|(A + E)^{-1}Ex\|,$$

where $\mathrm{vec}(E) \sim (0, \Sigma)$ follows various distributions. We attempt to quantify the following:

- How descriptive on average is the known worst-case analysis for small perturbations?
- Can anything be said about typical sensitivity of $x$ even for large perturbations?

We provide asymptotic results for $\|\Sigma\| \to 0$ as well as bounds that hold for large $\|\Sigma\|$. We extend some of our results to structured perturbations as well as to the full rank linear least squares problem.

## Matrices and Graph Theory

### Minimum rank, maximum nullity, and zero forcing number of simple digraphs

Adam Berliner, Minerva Catral, Leslie Hogben, My Huynh, Kelsey Lied, Michael Young

Extensive work has been done on problems related to finding the minimum rank among the family of real symmetric matrices whose off-diagonal zero-nonzero pattern is described by a given simple graph $G$. In this talk, we will discuss the case of simple digraphs, which describe the off-diagonal zero-nonzero pattern of a family of (not necessarily symmetric) matrices. Furthermore, we will establish cut-vertex reduction formulas for minimum rank and zero forcing number for simple digraphs, analyze the effect of deletion of a vertex on minimum rank and zero forcing number, and characterize simple digraphs whose zero forcing number is very low or very high.

### Equitable partitions and the normalized Laplacian matrix

Steve Butler

The normalized Laplacian matrix is $\mathcal{L} = D^{-1/2}(D - A)D^{-1/2}$. This matrix is connected to random walks and the eigenvalues of this matrix gives useful information about a graph, in some cases similar to what the eigenvalues of the adjacency or combinatorial Laplacian gives about a graph. We will review some of these basic properties and then focus on equitable partitions and their relationship to the eigenvalues of the normalized Laplacian. (An equitable partition is a partitioning of the vertices into $V_1 \cup V_2 \cdots \cup V_k$ sets such that the the number of edges between a vertex $v \in V_i$ and $V_j$ is independent of $v$.)

### Zero Forcing Number and Maximum Nullity of Subdivided Graphs

Minerva Catral, Leslie Hogben, My Huynh, Kirill Lazebnik, Michael Young, Anna Cepek, Travis Peters

Let $G = (V, E)$ be a simple, undirected graph with vertex set $V$ and edge set $E$. For an edge $e = uv$ of $G$, define $G_e$ to be the graph obtained from $G$ by inserting a new vertex $w$ into $V$, deleting the edge $e$ and inserting edges $uw$ and $wv$. We say that that the edge $e$ has been *subdivided* and call $G_e$ an *edge subdivision* of $G$. A *complete subdivision graph* $s(G)$ is obtained from a graph $G$ by subdividing every edge of $G$ once. In this talk, we investigate the maximum nullity and zero forcing number of graphs obtained by a finite number of edge subdivisions of a given graph, and present results that support the conjecture that for all graphs $G$, the maximum nullity of $s(G)$ and the zero forcing number of $s(G)$ coincide.

### The rank of a positive semidefinite matrix and cycles in its graph

Louis Deaett

Given a connected graph $G$ on $n$ vertices, consider an $n \times n$ positive semidefinite Hermitian matrix whose $ij$-entry is nonzero precisely when the $i$th and $j$th vertices of $G$ are adjacent, for all $i \neq j$. The smallest rank of such a matrix is the *minimum semidefinite rank* of $G$. If $G$ is triangle-free, a result of Moshe Rosenfeld gives a lower bound of $n/2$ on its minimum semidefinite rank. This talk presents a new proof of this result and uses it as an avenue to explore a more general connection between cycles in a graph and its minimum semidefinite rank.

### On the minimum number of distinct eigenvalues of a graph

Shaun Fallat

For a graph $G$, the minimum number of distinct eigenvalues, where the minimum is taken over all matrices in $S(G)$ is denoted by $q(G)$. This parameter is very important to understanding the inverse eigenvalue problem for graphs and, perhaps, has not received as much attention as its counterpart, the maximum multiplicity. In this talk, I will discuss graphs attaining extreme values of $q$ and will focus on certain families of graphs, including joins of graphs, complete bipartite graphs, and cycles. This work is joint with other members of the Discrete Math Research Group at the University of Regina.

### Refined Inertias of Tree Sign Patterns

Colin Garnett, Dale Olesky, Pauline van den Driessche

The refined inertia $(n_+, n_-, n_z, 2n_p)$ of a real matrix is the ordered 4-tuple that subdivides the number $n_0$ of eigenvalues with zero real part in the inertia $(n_+, n_-, n_0)$ into those that are exactly zero ($n_z$) and those that are pure imaginary ($2n_p$). For $n \geqslant 2$, the set of refined inertias $\mathbb{H}_n = \{(0, n, 0, 0), (0, n-2, 0, 2), (2, n-2, 0, 0)\}$ is important for the onset of Hopf bifurcation in dynamical systems. In this talk we discuss tree sign patterns of order $n$ that require or allow the refined inertias $\mathbb{H}_n$. We describe necessary and sufficient conditions for a tree sign pattern to require $\mathbb{H}_4$. We prove that if a star sign pattern requires $\mathbb{H}_n$, then it must have exactly one zero diagonal entry associated with a leaf in its digraph.

### The Minimum Coefficient of Ergodicity for a Markov Chain with a Specified Directed Graph

Steve Kirkland

Suppose that $T$ is an $n \times n$ stochastic matrix. The function $\tau(T) = \frac{1}{2}\max_{i,j=1,\ldots,n}\{||(e_i - e_j)^T T||_1\}$ is known as a coefficient of ergodicity for $T$, and measures the rate at which the iterates of a Markov chain with transition matrix $T$ converge to the stationary distribution vector. Many Markov chains are equipped with an underlying combinatorial structure that is described by a directed graph, and in view of that fact, we consider the following problem: given a directed graph $D$, find $\tau_{\min}(D) \equiv \min \tau(T)$, where the minimum is taken over all stochastic matrices $T$ whose directed graph is subordinate to $D$.

In this talk, we characterise $\tau_{\min}(D)$ as the solution to a linear programming problem. We then go on to provide bounds on $\tau_{\min}(D)$ in terms of the maximum outdegree of $D$ and the maximum indegree of $D$. A connection is established between the equality case in a lower bound on $\tau_{\min}(D)$ and certain incomplete block designs.

### The Dirac operator of a finite simple graph

Oliver Knill

The Dirac operator D of a finite simple graph G is a n x n matrix, where n is the total number of cliques in the graph. Its square L is the Laplace-Beltrami operator, a block diagonal matrix which leaves linear spaces of p-forms invariant. The eigenvalues of L come in pairs; each nonzero fermionic eigenvalue appears also as a bosonic eigenvalue. This super symmetry leads to the McKean Singer result that the super trace of the heat kernel str(exp(-tL)) is the Euler characteristic of G for all t. We discuss combinatorial consequences for the graph related to random walks and trees in the simplex graph and the use of D to compute the cohomology of the graph via the Hodge theorem. The matrix D is useful also for studying spectral perturbation problems or to study discrete partial differential equations like the wave equation u"=-Lu or heat equation u'=-Lu on the graph. We will show some animations of such evolutions.

### Logarithmic Tree Numbers for Acyclic Complexes

Woong Kook, Hyuk Kim

For a $d$-dimensional cell complex $\Gamma$ with $\tilde{H}_i(\Gamma) = 0$ for $-1 \leqslant i < d$, an $i$-dimensional tree is a non-empty collection $B$ of $i$-dimensional cells in $\Gamma$ such that $\tilde{H}_i(B \cup \Gamma^{(i-1)}) = 0$ and $w(B) := |\tilde{H}_{i-1}(B \cup \Gamma^{(i-1)})|$ is finite, where $\Gamma^{(i)}$ is the $i$-skeleton of $\Gamma$. Define the $i$-th tree-number to be $k_i := \sum_B w(B)^2$, where the sum is over all $i$-dimensional trees. In this talk, we will show that if $\Gamma$ is acyclic and $k_i > 0$ for $-1 \leqslant i \leqslant d$, then $k_i$ and the combinatorial Laplace operators $\Delta_i$ are related by $\sum_{i=-1}^{d} \omega_i \, x^{i+1} = (1+x)^2 \sum_{i=0}^{d-1} \kappa_i x^i$, where $\omega_i = \log\det \Delta_i$ and $\kappa_i = \log k_i$. We will discuss various applications of this equation.

### Minimum rank for circulant graphs

Seth Meyer

There are many known bounds on the minimum rank of a graph, the two we will focus on are the minimum semi-definite rank and the zero forcing number. While much is known about these parameters for small graphs, families of large graphs remain an object of study. In this talk we will discuss bounds on the minimum rank of graphs with circulant adjacency matrices, more commonly known as circulant graphs.

### Matrix Ranks in Graph Theory

T. S. Michael

Matrix ranks over various fields have several applications in graph theory and combinatorics—for instance, in establishing the non-existence of a graph or the non-isomorphism of two graphs. We discuss some old and new theorems along these lines with an emphasis on tournaments and graph decompositions.

### The Algebraic Connectivity of Planar Graphs

Jason Molitierno

The Laplacian matrix for a graph on $n$ vertices labeled $1, \ldots, n$ is the $n \times n$ matrix $L = [\ell_{I,j}]$ in which $\ell_{ii}$ is the degree of vertex $i$ and $\ell_{ij}$, for $i \neq j$, is -1 if vertices $i$ and $j$ are adjacent and 0 otherwise. Since $L$ is positive semidefinite and singular, we can order the eigenvalues $0 = \lambda_1 \leqslant \lambda_2 \leqslant \ldots, \leqslant \lambda_n$. The eigenvalue $\lambda_2$ is known as the algebraic connectivity of a graph as it gives a measure of how connected the graph is. In this talk, we derive an upper bound on the algebraic connectivity of planar graphs and determine all planar graphs in which the upper bound is achieved. If time permits, we will extend these results in two directions. First, we will derive smaller upper bounds on the algebraic connectivity of planar graphs with a large number of vertices. Secondly, we will consider upper bounds on the algebraic connectivity of a graph as a function of its genus.

### Using the weighted normalized Laplacian to construct cospectral unweighted bipartite graphs

Steven Osborne

The normalized Laplacian matrix of a simple graph $G$ is

$$\mathcal{L}(G)_{uv} := \begin{cases} 1 & \text{if } u = v \text{ and } \deg v \neq 0, \\ -\frac{1}{\sqrt{\deg u \deg v}} & \text{if } u \sim v, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

We say two graphs $G$ and $H$ are cospectral with respect to the normalized Laplacian if $\mathcal{L}(G)$ and $\mathcal{L}(H)$ have the same spectrum including multiplicity. Even for small $n$, it is difficult to build two graphs on $n$ vertices which are cospectral. We can extend the definition of the normalized Laplacian to weighted graphs. We show that the spectrum of certain weighted graphs is related to the spectrum of related unweighted graphs. This allows us to construct unweighted graphs which are cospectral with respect to the normalized Laplacian.

### The $\lambda$-$\tau$ problem for symmetric matrices with a given graph

Bryan Shader, Keivan Hassani Monfared

Let $\lambda_1 < \cdots < \lambda_n$, $\tau_1 < \cdots < \tau_{n-2}$ be $2n-2$ real numbers that satisfy the second order Cauchy interlacing inequalities $\lambda_i < \tau_i < \lambda_{i+2}$ and a nondegeneracy condition $\lambda_{i+1} \neq \tau_i$. Given a connected graph $G$ on $n$ vertices where vertices 1 and 2 are adjacent, it is proven that there is a real symmetric matrix $A$ with eigenvalues $\lambda_1, \ldots, \lambda_n$, and eigenvalues of $A(\{1,2\})$ are $\tau_1, \ldots, \tau_{n-2}$, such that graph of $A$ is $G$, provided each connected component of the graph obtained from $G$ by deleting the edge 1–2 has a sufficient number of vertices.

### A Simplified Principal Minor Assignment Problem

Pauline van den Driessche

Given a vector $u \in \mathrm{R}^{2^n}$, the principal minor assignment problem asks when is there a matrix of order $n$ having its $2^n$ principal minors (in lexicographic order) equal to $u$. Given a sequence $r_0r_1 \cdots r_n$ with $r_j \in \{0,1\}$, this simplified principal minor assignment problem asks when is there a real symmetric matrix of order $n$ having a principal submatrix of rank $k$ iff $r_k = 1$ for $0 \leqslant k \leqslant n$. Several families of matrices are constructed to attain certain classes of sequences, with particular emphasis on adjacency matrices of graphs. Other classes of sequences are shown to be not attainable. This is joint work with R.A. Brualdi, L. Deaett, D.D. Olesky.

### Generalized Propagation Time of Zero Forcing

Michael Young,

Zero forcing is a type of graph propagation that starts with a coloring of the vertices as white and blue and at each step any vertex colored blue with a unique neighbor colored white"forces" the color of the white vertex to be come blue. In this talk, we look at what happens to the length of time that it takes for all vertices to become blue when the size of the initial set of vertices colored blue is altered. We also discuss a tight relationship between the zero forcing number and the number of edges in the graph.

## Matrices and Orthogonal Polynomials

### Optimal designs, orthogonal polynomials and random matrices

Holger Dette

The talk explains several relations between different areas of mathematics: Mathematical statistics, random matrices and special functions. We give a careful introduction in the theory of optimal designs, which are used to improve the accuracy of statistical inference without performing additional experiments. It is demonstrated that for certain regression models orthogonal polynomials play an important role in the construction of optimal designs. In the next step these results are connected with some classical facts from random matrix theory.

In the third part of this talk we discuss some new results on special functions and random matrices. In particular we analyze random band matrices, which generalize the classical Gauschen ensemble. We show that the random eigenvalues of such matrices behave similarly as the deterministic roots of matrix orthogonal polynomials with varying recurrence coefficients. We study the asymptotic zero distribution of such polynomials and demonstrate that these results can be used to find the asymptotic properties of the spectrum of random band matrices.

### Finite-difference Gaussian rules

Vladimir Druskin

Finite-difference Gaussian rules (FDGR) a. k. a. optimal or spectrally matched grids were introduced for accurate approximation of Neumann-to-Dirichlet (NtoD ) maps of second order PDEs. The technique uses simple second order finite-difference approximations with optimized placement of the grid points yielding exponential superconvergence of the finite-difference NtoD. The fact that the NtoD map is well approximated makes the approach ideal for inverse problems and absorbing boundary conditions. We discuss connection of the FDGR to the Jacobi inverse eigenvalue problems, Stieltjes continued fractions, rational approximations and some other different classical techniques. Contributors: Sergey Asvadurov, Liliana Borcea, Murthy Guddatti, Fernando Guevara Vasquez, David Ingerman, Leonid Knizhnerman, Alexander Mamonov and Shari Moskow

### Wronskian type determinants of orthogonal polynomials, Selberg type formulas and constant term identities

Antonio Duran

Determinants whose entries are orthogonal polynomials is a long studied subject. One can mention Turán inequality for Legendre polynomials and its generalizations, specially that of Karlin and Szegő on Hankel determinants whose entries are ultraspherical, Laguerre, Hermite, Charlier, Meixner, Krawtchouk and other families of orthogonal polynomials. Karlin and Szegő's strategy was to express these Hankel determinants in terms of the Wronskian of certain orthogonal polynomials of another class. B. Leclerc extended this result by stating that a Wronskian of orthogonal polynomials is always proportional to a Hankel determinant whose elements form a new sequence of polynomials (not necessarily orthogonal). The purpose of this talk is to show a broad extension of this result for any linear operator $T$ acting in the linear space of polynomials and satisfying that $\deg(T(p)) = \deg(p) - 1$, for all polynomial $p$. In such a case, we establish a procedure to compute the Wronskian type $n \times n$ determinant $\det \left(T^{i-1}(p_{m+j-1}(x))\right)_{i,j=1}^{n}$, where $(p_n)_n$ is any sequence of orthogonal polynomials. For $T = d/dx$ we recover Leclerc's result, and for $T = \Delta$ (the first order difference operator) we get some nice symmetries for Casorati determinants of classical discrete orthogonal polynomials. We also show that for certain operators $T$, the procedure is related to some Selberg type integrals and constant term identities of certain multivariate Laurent expansions.

### Matrix Orthogonal Polynomials in Bivariate Orthogonal Polynomials.

Jeff Geronimo

The theory matrix orthogonal polynomials sheds much light on the theory of Bivariate orthogonal polynomials on the bicircle and on the square especially in the context of bivariate Bernstein-Szego measures. We will review the theory of matrix orthogonal polynomials and elaborate on the above connection.

### The tail condition for Leonard pairs

Edward Hanson

Let $V$ denote a vector space with finite positive dimension. We consider an ordered pair of linear transformations $A : V \to V$ and $A^* : V \to V$ that satisfy (i) and (ii) below:

(i) There exists a basis for $V$ with respect to which the matrix representing $A$ is irreducible tridiagonal and the matrix representing $A^*$ is diagonal.

(ii) There exists a basis for $V$ with respect to which the matrix representing $A^*$ is irreducible tridiagonal and the matrix representing $A$ is diagonal.

We call such a pair a *Leonard pair* on $V$. In this talk, we will discuss two characterizations of Leonard pairs that utilize the notion of a tail. This notion is borrowed from algebraic graph theory.

### Biradial orthogonal polynomials and Complex Jacobi matrices

Marko Huhtanen, Allan Peramaki

Self-adjoint antilinear operators have been regarded as yielding an analogue of normality in the antilinear case. This turns out to be the case also for orthogonal polynomials. Orthogonality based on a three term recurrence takes place on so-called biradial curves in the complex plane. Complex Jacobi matrices arise.

### Laurent orthogonal polynomials and a QR algorithm for pentadiagonal recursion matrices

Carl Jagels

The Laurent-Arnoldi process is an analog of the standard Arnoldi process applied to the extended Krylov subspace. It produces an orthogonal basis for the subspace along with a generalized Hessenberg matrix whose entries consist of the recursion coefficients. As in the standard case, the application of the process to certain types of linear operators results in recursion formulas with few terms. One instance of this occurs when the operator is Hermitian. This case produces an analog of the Lanczos process in which the accompanying orthogonal polynomials are now Laurent polynomials. The process applied to the approximation of matrix functions results in a rational approximation method that converges more quickly for an important class of functions than does the polynomial based Lanczos method. However the recursion matrix the process produces is no longer tridiagonal. It is pentadiagonal. This talk demonstrates that an analog of the QR algorithm applied to tridiagonal matrices exists for pentadiagonal recursion matrices and that the orthogonal matrix, $Q$, is a CMV matrix.

### Spectral theory of exponentially decaying perturbations of periodic Jacobi matrices

Rostyslav Kozhan

We fully characterize spectral measures of exponentially decaying perturbations of periodic Jacobi matrices. This gives numerous useful applications:

- We establish the "odd interlacing" property of the eigenvalues and resonances (the analogue of Simon's result for Schrödinger operators): between any two consecutive eigenvalues there must be an odd number of resonances.
- We solve the inverse resonance problem: eigenvalues and resonances uniquely determine the Jacobi matrix.
- We obtain the necessary and sufficient conditions on a set of points to be the set of eigenvalues and resonances of a Jacobi matrix.
- We study the effect of modifying an eigenmass or a resonance on the Jacobi coefficients.

Some of the related questions for the Jacobi matrices with matrix-valued coefficients will also be discussed.

### Scalar and Matrix Orthogonal Laurent Polynomials in the Unit Circle and Toda Type Integrable Systems

Manuel Manas, Carlos Alvarez Fernandez, Gerardo Ariznabarreta

The Gauss-Borel factorization of a semi-infinite matrix known as the Cantero-Morales-Velazquez moment matrix leads to the algebraic aspects of theory of orthogonal Laurent polynomials in the unit circle. In particular, we refer to determinantal expressions of the polynomials, five term recursion relations and Christoffel-Darboux formulas. At the same time the Gauss-Borel factorization of convenient "multi-time" deformations of the moment matrix leads to the integrable theory of the 2D Toda hierarchy. Hence, we explore how the integrable elements such as wave functions, Lax and Zakharov-Shabat equations, Miwa formalism, bilinear equations and tau functions are related to orthogonal Laurent polynomials in the unit circle. Finally, we extend the study to the matrix polynomial realm where now we have right and left matrix orthogonal Laurent polynomials.

The scalar part of this talk has been recently published in Advances in Mathematics **240** (2013) 132193.

### CMV matrices and spectral transformations of measures

Francisco Marcellan

In this lecture we deal with spectral transformations of nontrivial probability measures supported on the unit circle. In some previous work with L. Garza, when linear spectral transformations are considered, we have analyzed the connection between the corresponding Hessenberg matrices (GGT matrices) associated with the multiplication operator in terms of the orthonormal polynomial bases of the original and the transformed measures, respectively. The aim of our contribution is to consider the connection between the CMV matrices associated with the multiplication operator with respect to Laurent orthonormal bases corresponding to a nontrivial probability measure supported on the unit circle and a linear spectral transformation of it. We focus our attention on the Christoffel and Geronimus transformations where the QR and Cholesky factorizations play a central role. Notice that in the case of Jacobi matrices, which represent the multiplication operator in terms of an orthonormal basis, the analysis of such spectral transformations (direct and inverse Darboux transformations, respectively) yield LU and UL factorizations. A connection between Jacobi and Hessenberg matrices using the Szegő transformations of measures supported on the interval [-1,1] and some particular measures supported on the unit circle is also studied.

This is a joint work with M. J. Cantero, L. Moral, and L. Velázquez (Universidad de Zaragoza, Spain).

**Rational Krylov methods and Gauss quadrature**

Carl Jagels, Miroslav Pranic, Lothar Reichel, Xuebo Yu

The need to evaluate expressions of the form $f(A)v$ or $v^H f(A)v$, where $A$ is a large sparse or structured matrix, $v$ is a vector, $f$ is a nonlinear function, and $^H$ denotes transposition and complex conjugation, arises in many applications. Rational Krylov methods can be attractive for computing approximations of such expressions. These methods project the approximation problem onto a rational Krylov subspace of fairly small dimension, and then solve the small approximation problem so obtained. We are interested in the situation when the rational functions that define the rational Krylov subspace have few distinct poles. We discuss the case when $A$ is Hermitian and an orthogonal basis for the rational Krylov subspace can be generated with short recursion formulas. Rational Gauss quadrature rules for the approximation of $v^H f(A)v$ will be described. When $A$ is non-Hermitian, the recursions can be described by a generalized Hessenberg matrix. Applications to pseudospectrum computations are presented.

**Leonard pairs and the $q$-Tetrahedron algebra**

Paul Terwilliger

A Leonard pair is a pair of diagonalizable linear transformations of a finite-dimensional vector space, each of which acts in an irreducible tridiagonal fashion on an eigenbasis for the other one. The Leonard pairs are classified up to isomorphism, and correspond to the orthogonal polynomials from the terminating branch of the Askey scheme. The most general polynomials in this branch are the $q$-Racah polynomials. The $q$-Tetrahedron algebra was introduced in 2006. In this talk we show that each Leonard pair of $q$-Racah type gives a module for the $q$-Tetrahedron algebra. We discuss how this module illuminates the structure of the Leonard pair.

**A general matrix framework to predict short recurrences linked to (extended) Krylov spaces**

Raf Vandebril, Clara Mertens

In many applications, the problems to be solved are related to very large matrices. For system solving, as well as eigenvalue computations, researchers quite often turn to Krylov spaces to project onto, and as such reduce the size of the original problem to a feasible dimension. Unfortunately the complexity of generating a reliable basis for these Krylov spaces grows quickly, hence the search for short recurrences to compute one basis vector after the other in a modest amount of computing time.

Typically the search for short recurrences is based on inclusion theorems for the Krylov spaces and on the use of inner product relations, often resulting in quite complicated formulas and deductions. In this lecture we will predict the existence of short recurrences by examining the associated matrix structures. We will provide a general framework relying solely on matrix operations to rededuce many existing results. Focus will be on (extended, i.e., involving inverse matrix powers) Krylov spaces for Hermitian, Unitary, and normal matrices whose conjugate transpose is a rational function of the matrix itself. Based on this framework it will be possible to easily extract the underlying short recurrences of the basis vectors, constructed from the Krylov vectors.

**A quantum approach to Khrushchev's formula**

Francisco Grunbaum, Luis Velazquez, Albert Werner, Reinhard Werner

The beginning of the last decade knew a great advance in the theory of orthogonal polynomials on the unit circle due to the work of Sergei Khrushchev, who introduced many revolutionary ideas connecting with complex analysis and the theory of continued fractions. This body of techniques and results, sometimes known as Khrushchev's theory, has as a cornerstone the so called Khrushchev's formula, which identifies the Schur function of an orthogonal polynomial modification of a measure on the unit circle.

On the other hand, it was recently discovered that orthogonal polynomials on the unit circle play a significant role in the study of quantum walks, the quantum analogue of random walks. In particular the Schur functions, which are central to Khrushchev's theory, turn out to be the mathematical tool which best codifies the recurrence properties of a quantum walk. Indeed, Khrushchev's formula has proved to be an important tool for the study of quantum recurrence.

Nevertheless, this talk is not devoted to the applications of orthogonal polynomials on the unit circle or Khrushchev's theory to quantum walks, but the opposite. The most interesting connections use to be symbiotic, and this is one more example. The usefulness of Schur functions in quantum walks has led to a simple understanding of Khrushchev's formula using quantum matrix diagrammatic techniques. This goes further than giving a quantum meaning to a known mathematical result, but this kind of 'Feynman diagrammatic' provides very effective tools to generate new Khrushchev's type formulas, which end in splitting rules for scalar and matrix Schur functions.

### Harmonic oscillators and multivariate Krawtchouk polynomials

Luc Vinet

A quantum-mechanical model for the multivariate Krawtchouk polynomials of Rahman is given with the help of harmonic oscillators. It entails an algebraic interpretation of these polynomials as matrix elements of representations of the orthogonal groups. For simplicity, the presentation will focus on the 3-dimensional case where the bivariate Krawtchouk polynomials are associated to representations of the rotation group $SO(3)$ and their parameters related to Euler angles. Their characterization and applications will be looked at from this perspective.

(Based on joint work with Alexei Zhedanov.)

### Norm-constrained determinantal representations of polynomials

Anatolii Grinshpan, Dmitry Kaliuzhnyi-Verbovetskyi, Hugo Woerdeman

For every multivariable polynomial $p$, with $p(0) = 1$, we construct a determinantal representation

$$p = \det(I - KZ),$$

where $Z$ is a diagonal matrix with coordinate variables on the diagonal and $K$ is a complex square matrix. Such a representation is equivalent to the existence of $K$ whose principal minors satisfy certain linear relations. When norm constraints on $K$ are imposed, we give connections to the multivariable von Neumann inequality, Agler denominators, and stability. We show that if a multivariable polynomial $q$, $q(0) = 0$, satisfies the von Neumann inequality, then $1 - q$ admits a determinantal representation with $K$ a contraction. On the other hand, every determinantal representation with a contractive $K$ gives rise to a rational inner function in the Schur–Agler class.

## Matrices and Total Positivity

### Computations with matrices with signed bidiagonal decomposition

Alvaro Barreras, Juan Manuel Pena

We present a class of matrices with bidiagonal decompositions satisfying some sign restrictions. This class of matrices contains all nonsingular totally positive matrices as well as their inverses and opposite matrices. It inherits many properties of totally positive matrices. It is shown that one can compute to high relative accuracy theirs eigenvalues, singular values, inverses, triangular decompositions and the solution of some linear systems provided the bidiagonal decomposition. Backward stability of elimination procedures for solving linear systems is also analyzed.

### Quasi-$LDU$ factorization of totally nonpositive matrices.

Rafael Canto, Beatriz Ricarte, Ana Urbano

A matrix is called *totally nonpositive (totally negative)* if all its minors are nonpositive (negative) and is abbreviated as t.n.p. (t.n.). These matrices can be considered as a generalization of the partially negative matrices, that is, matrices with all its principal minors negative. The partially negative matrices are called N-matrices in economic models. If, instead, all minors of a matrix are nonnegative (positive) the matrix is called *totally positive(strictly totally positive)* and it is abbreviated as TP (STP).

The t.n.p. matrices with a negative $(1, 1)$ entry have been characterized using the factors of their $LDU$ factorization. For nonsingular t.n.p. matrices with the $(1, 1)$ entry equal to zero, a characterization is given in terms of the sign of some minors.

Now, we characterize the t.n.p. matrices with the $(1, 1)$ entry equal to zero in terms of a $\tilde{L}DU$ full rank factorization, where $\tilde{L}$ is a block lower echelon matrix, $U$ is a unit upper echelon TP matrix and $D$ is a diagonal matrix. This result holds for square t.n.p. matrices when the $(n, n)$ entry is equal to zero or when it is negative but we do not use permutation similarity. Finally, some properties of this kind of matrices are obtained.

### qd-type methods for totally nonnegative quasiseparable matrices

Roberto Bevilacqua, Enrico Bozzo, Gianna Del Corso

We present an implicit shifted $LR$ method for the class of quasiseparable matrices representable in terms of the parameters involved in their Neville factorization. The method resembles the qd method for tridiagonal matrices, and has a linear cost per step. For totally nonnegative quasiseparable matrices, it is possible to show that the algorithm is stable and breakdown cannot occur when the Laguerre shift, or another strategy preserving nonnegativity, is used.

**Accurate and fast algorithms for some totally nonnegative matrices arising in CAGD**

Jorge Delgado

Gasca and Peña showed that nonsingular totally nonnegative (TNN) matrices and its inverses admit a bidiagonal decomposition. Koev, in a recent work, assuming that the bidiagonal decompositions of a TNN matrix and its inverse are known with high relative accuracy (HRA), presented algorithms for performing some algebraic computations with high relative accuracy: computing the eigenvalues and the singular values of the TNN matrix, computing the inverse of the TNN matrix and obtaining the solution of some linear systems whose coefficient matrix is the TNN matrix.

Rational Bernstein and Said-Ball bases are usual representations in Computer Aided Geometric Design (CAGD). In some applications in this field it is necessary to solve some of the algebraic problems mentioned before for the collocations matrices of those bases (RBV and RSBV matrices, respectively). In our talk we will show, taking into account that RBV and RSBV matrices are TNN, how to compute the bidiagonal decomposition of the RBV and RSBV matrices with HRA. Then we will apply Koevs algorithms showing the accuracy of the obtained results for the considered algebraic problems.

**The Totally Positive (Nonnegative) Completion Problem and Other Recent Work**

Charles Johnson

We survey work on the TP completion problem for ”connected” patterns.
Time permitting, we also mention some recent work on perturbation of TN to TP. This relates the possible numbers and arrangements of of equal entries in a TP matrix to possible incidences of points and lines in the plane, a most exciting advance.

**Generalized total positivity and eventual properties of matrices**

Olga Kushel

Here we study the following classes of matrices: eventually totally positive matrices, Kotelyansky matrices and eventually P-matrices. We analyze the relations between these three classes. We also discuss common spectral properties of matrices from these classes. The obtained results are applied to some spectral problems.

**Deleting derivations algorithm and TNN matrices**

Stephane Launois

In this talk, I will present the deleting derivations algorithm, which was first developed in the context of quantum algebras, and explain the significance of this algorithm for the study of TNN matrices.

**More accurate polynomial least squares fitting by using the Bernstein basis**

Ana Marco, José-Javier Martínez

The problem of least squares fitting with algebraic polynomials of degree less than or equal to $n$, a problem with applications to approximation theory and to linear regression in statistics, is considered.

The coefficient matrix $A$ of the overdetermined linear system to be solved in the least squares sense is a rectangular Vandermonde matrix if the monomial basis is used for the space of polynomials or a rectangular Bernstein-Vandermonde matrix if the Bernstein basis is used. Under certain conditions both classes of matrices are totally positive, but they usually are ill-conditioned matrices (a problem related to the collinearity subject in statistical regression), which leads to a loss of accuracy in the solution.

Making use of algorithms for the $QR$ factorization of totally positive matrices the problem is more accurately solved, indicating how to use the Bernstein basis for a general problem and taking advantage of the fact that the condition number of a Bernstein-Vandermonde matrix is smaller than the condition number of the corresponding Vandermonde matrix.

## Matrices over Idempotent Semirings

**Tolerance and strong tolerance interval eigenvectors in max-min algebra**

Martin Gavalec, Jan Plavka

In most applications, the vector and matrix inputs are not exact numbers and they can rather be considered as values in some interval. The classification and the properties of matrices and their eigenvectors, both with interval coefficients in max-min algebra, are studied in this contribution. The main result is the complete description of the eigenspace structure for the tolerance and the strong tolerance interval eigenproblem in max-min algebra. Polynomial algorithms are also suggested for solving the recognition version of both problems. Finally, the connection between the two interval eigenproblems is described and illustrated by numerical examples. The work has been prepared under the support of the Czech Science Foundation project 402/09/0405 and APVV 0008-10.

**The robustness of interval matrices in max-min algebra**

Jan Plavka, Martin Gavalec

The robustness of interval fuzzy matrices is defined as a sequence matrix powers and matrix-vector multiplication performed in max-min arithmetics ultimately which stabilizes for all matrices or at least for one matrix from a given interval. Robust interval matrices in max-min algebra and max-plus algebra describing discrete-event dynamic systems are studied and equivalent conditions for interval matrices to be robust are presented. Polynomial algorithms for checking the necessary and sufficient conditions for robustness of interval matrices are introduced. In addition, more efficient algorithms for verifying the robustness of interval circulant and interval zero-one fuzzy matrices are described. The work has been prepared under the support of the Czech Science Foundation project 402/09/0405 and the Research and Development Operational Program funded by the ERDF (project number: 26220120030).

### Bounds of Wielandt and Schwartz for max-algebraic matrix powers

Hans Schneider, Sergey Sergeev

By the well-known bound of Wielandt, the sequence of powers of primitive Boolean matrices becomes periodic after $(n-1)^2+1$, where $n$ is the size of the matrix or, equivalently, the number of nodes in the associated graph. In the imprimitive case, there is a sharper bound due to Schwartz making use of the cyclicity of the graph. We extend these bounds to the periodicity of columns and rows of max-algebraic powers with indices in the critical graph. This is our joint work with G. Merlet and T. Nowak.

### Gaussian elimination and other universal algorithms

Sergei Sergeev

Linear algebra over general semirings is a known area of pure mathematics. Due to applications of idempotent semirings in optimization, timetable design and other fields we are interested in building a more algorithmic and applied theory on the same level of abstraction. We will present some universal algorithms solving Z-matrix equations over semirings (based on joint work with G.L. Litvinov, V.P. Maslov and A.N. Sobolevskii) and discuss a semiring generalization of the Markov chain tree theorem (based on joint work with B.B. Gursoy, S. Kirkland and O. Mason).

## Matrix Inequalities

### Tsallis entropies and matrix trace inequalities in Quantum Statistical Mechanics

Natalia Bebiano

Extensions of trace inequalities arising in Statistical Mechanics are derived in a straightforward and unified way from a variational characterization of the Tsallis entropy. Namely, one-parameter extension of the thermodynamic inequality is presented and its equivalence to a generalized Peierls-Bogoliubov inequality is stated, in the sense that one may be obtained from the other, and vice-versa.

### An order-like relation induced by the Jensen inequality

Tomohiro Hayashi

In this talk I will consider a certain order-like relation for positive operators on a Hilbert space. This relation is defined by using the Jensen inequality with respect to the square-root function. I will explain some properties of this relation.

### Monomial Inequalities for p-Newton Matrices and Beyond

Charles Johnson

An ordered sequence of real numbers is Newton if the square of each element is at least the product of its two immediate neighbors, and p-Newton if, further, the sequence is positive. A real matrix is Newton (p-Newton) if the sequence of averages of the k-by-k principal minors is Newton (p-Newton). Isaac Newton is credited with the fact symmetric matrices are Newton. Of course, a number of determinantal inequalities hold for p-Newton matrices. We are interested in exponential polynomial inequalities that hold for all p-Newton sequences. For two monomials M1 and M2 in n variables, we determine which pairs satisfy M1 at least M2 for all p-Newton sequences. Beyond this, we discuss what is known about inequalities between linear combinations of monomials. These tend to be characterized by highly combinatorial statements about the indices in the monomials.

### A numerical radius inequality involving the generalized Aluthge transform

Fuad Kittaneh, Amer Abu Omar

A spectral radius inequality is given. An application of this inequality to prove a numerical radius inequality that involves the generalized Aluthge transform is also provided. Our results improve earlier results by Kittaneh and Yamazaki.

### Fischer type determinantal inequalities for accretive-dissipative matrices

Minghua Lin

The well known Fischer determinantal inequality for positive definite matrix says that the determinant of a partitioned matrix is bounded by the product of the determinant of its diagonal blocks. We investigate this type of inequality when the underlying matrix is accretive-dissipative, that is, the matrix whose real and imaginary part (in the Cartesian decomposition) are both positive definite.

### Two Conjectured Inequalities motivated by Quantum Structures.

Rajesh Pereira, Corey O'Meara

We show how some mathematical tools used in Quantum Theory give rise to some natural questions about inequalities. We look at interrelationships between certain subsets of completely positive linear maps and their applications to results on correlation matrices, Schur products and Grothendieck's inequalities. We also show interconnections between density matrices and the permanent on top conjecture. These connections are used to formulate a Quantum permanent on top conjecture which generalizes the original conjecture.

### Kwong matrices and related topics

Takashi Sano

In this talk, after reviewing our study on Loewner and Kwong (or anti-Loewner) matrices, we will show our recent results on them or generalized ones.

### Characterizations of some distinguished subclasses of normal operators by operator inequalities

Ameur Seddik

In this note, we shall present some characterizations of some distinguished subclasses of normal operators (self-adjoint operators, unitary operators, unitary reflection operators, normal operators) by operator inequalities.

### On Ky Fan's result on eigenvalues and real singular values of a matrix

Tin-Yau Tam, Wen Yan

Ky Fan's result states that the real parts of the eigenvalues of an $n \times n$ complex matrix $A$ is majorized by the real singular values of $A$. The converse was established independently by Amir-Moéz and Horn, and Mirsky. We extend the results in the context of complex semisimple Lie algebras. The real semisimple case is also discussed. The complex skew symmetric case and the symplectic case are explicitly worked out in terms of inequalities. The symplectic case and the odd dimensional skew symmetric case can be stated in terms of weak majorization. The even dimensional skew symmetric case involves Pfaffian.

### On some matrix inequalities involving matrix geometric mean of several matrices

Takeaki Yamazaki

In the recent years, the study of geometric means of several matrices is developing very much. Especially, the one of the geometric means is called the Karcher mean. It has many good properties. In this talk, we shall introduce some matrix inequalities which are relating to the Karcher mean. Results are extensions of known results, for example, Furuta inequality, Ando-Hiai inequality and famous characterization of chaotic order. Recently, the Karcher mean is considered as a limit of power mean defined by Lim-Pálfia. We will also introduce some matrix inequalities related to power mean.

### Some Inequalities involving Majorization

Fuzhen Zhang

We start with the discussions on angles of vectors and apply majorization to obtain some inequalities of vector angles of majorization type. We then study the majorization polytope of integral vectors. We present several properties of the cardinality function of the polytope. First part is a joint work with D. Castano and V.E. Paksoy (Nova Southeastern University, Florida, USA), and the second part is with G. Dahl, (University of Oslo, Norway).

## Matrix Methods for Polynomial Root-Finding

### A Factorization of the Inverse of the Shifted Companion Matrix

Jared Aurentz

A common method for computing the zeros of an $n^{th}$ degree polynomial is to compute the eigenvalues of its companion matrix $C$. A method for factoring the shifted companion matrix $C - \rho I$ is presented. The factorization is a product of $2n - 1$ essentially $2 \times 2$ matrices and requires $O(n)$ storage. Once the factorization is computed it immediately yields factorizations of both $(C - \rho I)^{-1}$ and $(C - \rho I)^*$. The cost of multiplying a vector by $(C - \rho I)^{-1}$ is shown to be $O(n)$ and therefore any shift and invert eigenvalue method can be implemented efficiently. This factorization is generalized to arbitrary forms of the companion matrix.

### A condensed representation of generalized companion matrices

Roberto Bevilacqua, Gianna Del Corso, Luca Gemignani

In this talk we give necessary and sufficient condition for a matrix such that $A^H = p(A) + C$ to be reduced to block tridiagonal form, where the size of the blocks depends on the rank of the correction matrix $C$. We show how to unitary reduce the matrix to this condensed form applying the block-Lanczos process to a suitable matrix starting form a basis of the space spanned by the columns of the matrix $CA - AC$. Among this class of matrices, that we called almost normal, we find companion matrices, comrade and colleagues matrices.

### Multiplication of matrices via quasiseparable generators and eigenvalue iterations for block trian-

**gular matrices**

Yuli Eidelman

We study special multiplication algorithms for matrices represented in the quasiseparable form. Such algorithms appear in a natural way in the design of iteration eigenvalue solvers for structured matrices. We discuss a general method which may be used as a main building block in the design of fast LR-type methods for upper Hessenberg or block triangular matrices. The comparison with other existing methods is performed and the results of numerical tests are presented.

### The Ehrlich-Aberth method for structured eigenvalue refinement

Luca Gemignani

This talk is concerned with the computation of a structured approximation of the spectrum of a structured eigenvalue problem by means of a root-finding method. The approach discussed here consists in finding the structured approximation by using the Ehrlich-Aberth algorithm applied for the refinement of an unstructured approximation providing a set of initial guesses. Applications to T-palindromic, H-palindromic and even/odd eigenvalue problems will be presented. This work is partly joint with S. Brugiapaglia and V. Noferini.

### Generalized Hurwitz matrices and forbidden sectors of the complex plane

Olga Holtz, Sergey Khrushchev, Olga Kushel, Mikhail Tyaglov

We investigate polynomials whose generalized Hurwitz matrices (to be defined in the talk) are totally nonnegative. Using a generalized Euclidean algorithm, we establish that their zeros avoid specific sectors in the complex plane. Multiple root interlacing turns out to be ultimately related to this phenomenon as well, although that connection is not yet completely understood. Finally, we suggest some generalizations and open problems.

### A generalized companion method to solve systems of polynomials

Matthias Humet, Marc Van Barel

A common method to compute the roots of a univariate polynomial is to solve for the eigenvalues of its companion matrix. If the polynomial is given in a Chebyshev basis, one refers to this matrix as the colleague matrix. By generalizing this procedure for multivariate polynomials, we are able to solve a system of n-variate polynomials by means of an eigenvalue problem. We obtain an efficient method that yields numerically accurate solutions for well-behaved polynomial systems. An advantage of the method is that it can work with any polynomial basis.

### Real and Complex Polynomial Root-finding by Means of Eigen-solving

Victor Pan

Our new numerical algorithms approximate real and complex roots of a univariate polynomial lying near a selected point of the complex plane, all its real roots, and all its roots lying in a fixed half-plane or in a fixed rectangular region. The algorithms seek the roots of a polynomial as the eigenvalues of the associated companion matrix. Our analysis and experiments show their efficiency. We employ some advanced machinery available for matrix eigen-solving, exploit the structure of the companion matrix, and apply randomized matrix algorithms, repeated squaring, matrix sign iteration and subdivision of the complex plane. Some of our techniques can be of independent interest.

### Solving secular and polynomial equations: a multiprecision algorithm

Dario Bini, Leonardo Robol

We present a numerical algorithm to approximate all the solutions of the secular equation $\sum_{i=1}^{n} \frac{a_i}{x-b_i} - 1 = 0$ with any given number of guaranteed digits. Two versions are provided. The first relies on the MPSolve approach of D. Bini and G. Fiorentino [Numer. Algorithms 23, 2000], the second on the approach of S. Fortune [J. of Symbolic Computation 33, 2002] for computing polynomial roots. Both versions are based on a rigorous backward error analysis and on the concept of root neighborhood combined with Gerschgorin theorems, where the properties of diagonal plus rank-one matrices are exploited.

An implementation based on the GNU multiprecision package (GMP) is provided which is included in the release 3.0 of the package MPSolve. It is downloadable from `http://riccati.dm.unipi.it/mpsolve`. The new release computes both the solutions of a secular equation and the roots of a polynomial assigned in terms of its coefficients. From the many numerical experiments, it turns out that the new release is generally much faster than MPSolve 2.0 and of the package Eigensolve of S. Fortune (`http://ect.bell-labs.com/who/sjf/eigensolve.html`). For certain polynomials, like the Mandelbrot or the partition polynomials, the acceleration is dramatic.

Comparison with techniques based on rank-structured matrices are performed. The extensions to computing eigenvalues of matrix polynomials and roots of scalar polynomials represented in different basis are discussed.

### The Fitting of Convolutional Models for Complex Polynomial Root Refinement

Peter Strobach

In the technical sciences, there exists a strong demand for high-precision root finding of univariate polynomials with real or complex coefficients. One can distinguish between cases of low-degree polynomials with high coefficient dynamics (typically "Wilkinson-like" polynomials) and high-degree low coefficient variance polynomials like random coefficient polynomials. The latter prototype case is typically found in signal and array processing, for instance, in the analysis of long z-transform polynomials of signals buried in noise, speech processing, or in finding the roots of z-transfer functions of high-degree FIR filters. In these areas, the polynomial degrees $n$ can be excessively high ranging up to $n = 10000$ or higher. Multiprecision software is practically not applicable in these cases, because of the $O(n^2)$ complexity of the problem and the resulting excessive runtimes. Established double precision root-finders are sufficiently fast but too inaccurate.

We demonstrate that the problem can be solved in standard double precision arithmetic by a two-stage procedure consisting of a fast standard double precision root finder for computing "coarse" initial root estimates, followed by a noniterative root refinement algorithm based on the fitting of a convolutional model onto the given "clean" coefficient sequence. We exploit the fact that any degree $n$ coefficient sequence can be posed as the convolution of a linear factor (degree 1) polynomial (effectively representing a root) and a corresponding degree $n-1$ subpolynomial coefficient sequence representing the remaining roots. Once a coarse estimate of a root is known, the corresponding degree $n-1$ subpolynomial coefficient sequence is simply computed as the solution of a least squares problem. Now both the linear factor and the degree $n-1$ subpolynomial coefficient sequences are *approximately* known. Using this information as input, we can finally fit both the linear factor and the degree $n-1$ subpolynomial coefficients *jointly* onto the given "clean" degree $n$ coefficient sequence, which is a nonlinear approximation problem. This concept amounts the evaluation of a sparse Jacobian matrix. When solved appropriately, this results in a complex orthogonal filter type algorithm for recomputing a given root estimate with substantially enhanced numerical accuracy.

Simulation results are shown for several types of polynomials which typically occur in signal and array processing. For instance, random coefficient polynomials up to a degree of $n = 64000$, where we reduce the absolute root errors of a standard double precision root finder by a factor of approximately 1000 (or 3 additional accurate mantissa digits in standard double precision arithmetic), or the roots of a complex chirp polynomial of degree $n = 2000$, where we reduced the absolute root errors by a factor of approximately 10000 using the new method.

**Companion pencil vs. Companion matrix: Can we ameliorate the accuracy when computing roots?**

Raf Vandebril

The search for accurate and reliable methods for fastly retrieving roots of polynomials based on matrix eigenvalue computations has not yet ended. After the first attempts based on suitable modifications of the QR algorithm applied on companion matrices trying to keep possession of the structure as much as possible, we have seen a recent shift of attention towards nonunitary LR algorithms and pencils.

In this lecture we will investigate the possibilities of developing reliable QZ algorithms for rootfinding. We thus step away from the classical companion setting and turn to the companion pencil approach. This creates more flexibility in the distribution of the polynomial coefficients and allows us an enhanced balancing of the associated pencil. Of course we pay a price: computing time and memory usage. We will numerically compare this novel QZ algorithm with a former QR algorithm in terms of accuracy, balancing sensitivity, speed and memory consumption.

**Fast computation of eigenvalues of companion, comrade, and related matrices**

Jared Aurentz, Raf Vandebril, David Watkins

The usual method for computing the zeros of a polynomial is to form the companion matrix and compute its eigenvalues. In recent years several methods that do this computation in $O(n^2)$ time with $O(n)$ memory by exploiting the structure of the companion matrix have been proposed. We propose a new class of methods of this type that utilizes a factorization of the comrade (or similar) matrix into a product of essentially $2 \times 2$ matrices times a banded upper-triangular matrix. Our algorithm is a non-unitary variant of Francis's implicitly shifted $QR$ algorithm that preserves this factored form. We will present numerical results and compare our method with other methods.

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**Matrix Methods in Computational Systems Biology and Medicine**

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**Two-Fold Irreducible Matrices: A New Combinatorial Class with Applications to Random Multiplicative Growth**

Lee Altenberg

Two-Fold Irreducible (TFI) matrices are a new combinatorial class intermediate between primitive and fully indecomposable matrices. They are the necessary and sufficient class for strict log-convexity of the spectral radius $r(e^D A)$ with respect to nonscalar variation in diagonal matrix $D$. Matrix $A$ is TFI when $A$ or $A^2$ is irreducible and $A^\top A$ or $AA^\top$ is irreducible, in which case $A$, $A^2$, $A^\top A$, and $AA^\top$ are all irreducible, and equivalently, the directed graph and simple bipartite graph associated with $A$ are each connected. Thus TFI matrices are the intersection of irreducible and chainable matrices. A Markov chain is TFI if and only if it is ergodic and running the process alternately forward and backward is also ergodic. The problem was motivated by a question from Joel E. Cohen who discovered that a stochastic process $N(t) = s_t \dots s_2 s_1 N(0)$ approaches Taylor's law in the limit of large $t$: $\lim_{t\to\infty}[\log \mathrm{Var}(N(t)) - b \log \mathrm{E}(N(t))] = \log a$, for some $a > 0$ and $b$, when $D$

is nonscalar, the spectral radius of $DP$ is not 1, and the transition matrix $P$ for the finite Markov chain $(s_1, s_2, s_3, ...)$ meets conditions needing to be characterized. The requisite condition is that $P$ be two-fold irreducible.

### Understanding real-world systems using random-walk-based approaches for analyzing networks

Natasa Durdevac, Marco Sarich, Sharon Bruckner, Tim Conrad, Christof Schuette

Real-world systems are often modeled as networks. Many algorithms for analyzing such complex networks are oriented towards (1) finding modules (or clusters, communities) that are densely inter-connected substructures having sparse connections to the rest of the network, and (2) finding hubs that are key connectors of modules. In many cases modules and hubs correspond to relevant structures in the original system, so their identification is important for understanding the organization of the underlying real-world system. For example in biological systems modules often correspond to functional units (like protein complexes) and hubs to essential parts of this system (like essential proteins).

In this talk, we will present a novel spectral-based approach for efficient fuzzy network partitioning into modules. We will consider random walk processes on undirected networks, for which modules will correspond to metastable sets: regions of the network where a random walker stays for a long time before exiting. In contrast to standard spectral clustering approaches, we will define different transition rules of the random walk, which will lead to much more prominent gaps in the spectrum of the adapted random walk process. This allows easier identification of network's modular structures and resolves the problem of alternating, cyclic structures identified as modules, improving upon previous methods.

The mathematical framework that will be presented in this talk includes solving several interesting linear algebra problems.

### Stochastic Dimension Reduction Via Matrix Factorization in Genomics and Proteomics

Amir Niknejad

Since the last decade the matrix factorization and dimension reduction methods have been used for mining in genomics and proteomics. In this talk we cover the basics of DNA Microarrays technology and dimension reduction techniques for analysis of gene expression data. Also, we discuss how biological information in gene expression matrix could be extracted, using a sampling framework along with matrix factorization incorporating rank approximation of gene matrix.

### Improved molecular simulation strategies using matrix analysis

Marcus Weber, Konstantin Fackeldey

According to the state of the art, complex molecular conformation dynamics can be expressed by Markov State Models. The construction of these models usually needs a lot of short time trajetory simulations. In this talk, methods based on linear algebra and matrix analysis are presented which help to reduce the computational costs of molecular simulation.

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## Multilinear Algebra and Tensor Decompositions

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### Between linear and nonlinear: numerical computation of tensor decompositions

Lieven De Lathauwer, Laurent Sorber, Marc Van Barel

In numerical multilinear algebra important progress has recently been made. It has been recognized that tensor product structure allows a very efficient storage and handling of the Jacobian and (approximate) Hessian of the cost function. On the other hand, multilinearity allows global optimization in (scaled) line and plane search. Although there are many possibilities for decomposition symmetry and factor structure, these may be conveniently handled. We demonstrate the algorithms using Tensorlab, our recently published MATLAB toolbox for tensors and tensor computations.

### Alternating minimal energy methods for linear systems in higher dimensions. Part II: Faster algorithm and application to nonsymmetric systems

Sergey Dolgov, Dmitry Savostyanov

In this talk we further develop and investigate the rank-adaptive alternating methods for high-dimensional tensor-structured linear systems. We introduce a recurrent variant of the ALS method equipped with the basis enrichment, which performs a subsequent linear system reduction, and seems to be more accurate in some cases than the method based on a global steepest descent correction. In addition, we propose certain heuristics to reduce the complexity, and discuss how the method can be applied to nonsymmetric systems. The efficiency of the technique is demonstrated on several examples of the Fokker-Planck and chemical master equations.

### Alternating minimal energy methods for linear systems in higher dimensions. Part I: SPD systems.

Sergey Dolgov, Dmitry Savostyanov

We introduce a family of numerical algorithms for the solution of linear system in higher dimensions with the matrix and right hand side given and the solution sought in the tensor train format. The proposed methods are rank–adaptive and follow the alternating directions framework, but in contrast to ALS methods, in each iteration a tensor subspace is enlarged by a set of vectors chosen similarly to the steepest descent algorithm. The convergence is analyzed in the presence of approximation errors and the geometrical convergence rate is estimated and related to the one of the steepest descent. The complexity of the presented algorithms is linear in the mode size and dimension and the convergence demonstrated in the numerical experiments is comparable to the one of the DMRG–type algorithm. Numerical comparison with conventional tensor methods is provided.

**The number of singular vector tuples and the uniqueness of best $(r_1, \ldots, r_d)$ approximation of tensors**

Shmuel Friedland

In the first part of the talk we give the exact number of singular tuples of a complex $d$-mode tensor. In the second part of the talk we show that for a generic real $d$-mode tensor the best $(r_1, \ldots, r_d)$ approximation is unique. Moreover for a generic symmetric $d$-mode tensor the best rank one approximation is unique, hence symmetric. Similar results hold for partially symmetric tensors. The talk is based on a recent joint paper with Giorgio Ottaviani and a recent paper of the speaker.

**From tensors to wavelets and back**

Vladimir Kazeev

The *Tensor Train* (TT) decomposition was introduced recently for the low-parametric representation of high-dimensional tensors. Among the tensor decompositions based on the separation of variables, the TT representation is remarkable due to the following two key features. First, it relies on the successive low-rank factorization (exact or approximate) of certain matrices related to the tensor to be represented. As a result, the low-rank approximation crucial for basic operations of linear algebra can be implemented in the TT format with the use of standard matrix algorithms, such as SVD and QR. Second, in many practical cases the particular scheme of separation of variables, which underlies the TT representation, does ensure linear (or almost linear) scaling of the memory requirements and of the complexity with respect to the number of dimensions.

Each dimension of a tensor can be split into a few virtual levels representing different scales within the dimension. Such a transformation, referred to as *quantization*, results in a tensor with more dimensions, smaller mode sizes and the same entries. The TT decomposition, being applied to a tensor after quantization, is called *Quantized Tensor Train* (QTT) decomposition. By attempting to separate all virtual dimensions and approximate the hierarchy of the scales, the QTT representation seeks more structure in tensors, compared to the TT format without quantization.

Consider the approximation of a given tensor in the TT format. Even when the low-rank matrix factorization mentioned above is performed crudely, i.e. with too small ranks, the factors carry a certain description of the tensor being approximated. The most essential TT "components", which are represented by those factors, can be filtered out from other tensors with the hope that the images are going to be sparse. Oseledets and Tyrtyshnikov proposed this approach under the name *Wavelet Tensor Train (WTT) transform* and showed the transform to be linear and orthogonal. Having been constructed with the use of quantization, it extracts the hierarchy of virtual levels in terms of that of the reference tensor and can be seen as an algebraic wavelet-type transform.

Our contribution consists in closing the "tensor-wavelet loop" by analyzing the tensor structure of the WTT transform. We give an explicit TT decomposition of its matrix in terms of that of the reference tensor. In particular, we establish a relation between the TT ranks of the two. Also, for a tensor given in the TT format we construct an explicit TT representation of its WTT image and bound the TT ranks of the latter. Finally, we show that the matrix of the WTT transform is sparse.

In short, the matrix of the WTT transform turns out to be a sparse, orthogonal matrix with low-rank TT structure, adapted in a certain way to a given vector.

This is a joint work with Ivan Oseledets (INM RAS, Moscow).

**Tensor-based Regularization for Multienergy X-ray CT Reconstruction**

Oguz Semerci, Ning Hao, Misha Kilmer, Eric Miller

The development of energy selective, photon counting X-ray detectors makes possible a wide range of possibilities in the area of computed tomographic image formation. Under the assumption of perfect energy resolution, we propose a tensor-based iterative algorithm that simultaneously reconstructs the X-ray attenuation distribution for each energy. We model the multi-spectral unknown as a 3-way tensor where the first two dimensions are space and the 3rd dimension is energy. We propose a tensor nuclear norm regularizer based on the tensor-SVD of Kilmer and Martin (2011) which is a convex function of the multi-spectral unknown. Simulation results show that the tensor nuclear norm can be used as a stand alone regularization technique for the energy selective (spectral) computed tomography (CT) problem and can be combined with additional spatial regularization for high quality image reconstructions.

**Multiplying higher-order tensors**

Lek-Heng Lim, Ke Ye

Two $n \times n$ matrices may be multiplied to obtain a third $n \times n$ matrix in the usual way. Is there a multiplication rule that allows one to multiply two $n \times n \times n$ hypermatrices to obtain a third $n \times n \times n$ hypermatrix? It depends on what we mean by multiplication. Matrix multiplication comes from composition of linear operators and is a coordinate independent operation. Linear operators are of course 2-tensors and matrices are their coordinate representations. If we require our multiplication of $n \times n \times n$-hypermatrices to come from multiplication of 3-tensors, then we show that the answer is no. In fact, we will see that two $d$-tensors can be multiplied to yield a third $d$-tensor if and only if $d$ is even, and in which case, the multiplication rule is essentially given by tensor contractions.

### Wrestling with Alternating Least Squares

Martin Mohlenkamp

Alternating Least Squares is the simplest algorithm for approximating by a sum of rank-one tensors. Yet even it, even on small problems, has (unpleasantly) rich behavior. I will describe my efforts to understand what it is doing and why.

### Tensor decompositions and optimization problems

Eugene Tyrtyshnikov

Tensor decompositions, especially Tensor Train (TT) and Hierarchical Tucker (HT), are fastly becoming useful and widely used computational instruments in numerical analysis and numerous applications. As soon as the input vectors are presented in the TT (HT) format, basic algebraic operations can be efficiently implemented in the same format, at least in a good lot of practical problems. A crucial thing is, however, to acquire the input vectors in this format. In many cases this can be accomplished via the TT-CROSS algorithm, which is a far-reaching extension of the matrix cross interpolation algorithms. We discuss some properties of the TT-CROSS that allow us to adopt it for the needs of solving a global optimization problem. After that, we present a new global optimization method based on special transformations of the scoring functional and TT decompositions of multi-index arrays of values of the scoring functional.

We show how this new method works in the direct docking problem, which is a problem of accommodating a ligand molecule into a larger target protein so that the interaction energy is minimized. The degrees of freedom in this problem amount to several tens. Most popular techniques are genetic algorithms, Monte Carlo and molecular dynamic approach. We have found that the new method can be up to one hundred times faster on typical protein-ligand complexes.

### On convergence of ALS and MBI for approximation in the TT format

Andre Uschmajew

We consider the alternating least squares algorithm (ALS) and the recently proposed maximum block improvement method (MBI, SIAM J. Optim., 22 (2012), pp. 87–107) for optimization with TT tensors. For both, a rank condition on the Hessian of the cost function can be imposed to prove local convergence. We review these conditions and derive singular value gap conditions for the approximated tensor which ensure them. Additionally, the MBI method has the advantage of producing globally convergent subsequences to critical points under the condition of boundedness. Both methods shall be compared in their numerical performance.

### Properties of the Higher-Order Generalized Singular Value Decomposition

Charles Van Loan, Orly Alter

The higher-order SVD is a way of simultaneously reducing each matrix in a collection $\{A_1, , A_N\}$ to an SVD-like form that permits one to identify common features. Each $A_i$ has the same number of columns. The invariant subspace associated with the minimum eigenvalue of the matrix $(S_1 + \cdots + S_n)(S_1^{-1} + \cdots + S_N^{-1})$ is involved where $S_i = A_i^T A_i$. We are able to compute this subspace stably by using a higher-order CS decomposition, a new reduction that we shall present in the talk. The behavior of the overall procedure when various error tolerances are invoked will be discussed as will connections to PARAFAC2.

### A Preconditioned Nonlinear Conjugate Gradient Algorithm for Canonical Tensor Decomposition

Manda Winlaw, Hans De Sterck

The alternating least squares (ALS) algorithm is currently the workhorse algorithm for computing the canonical tensor decompostion. We propose a new algorithm that uses ALS as a nonlinear preconditioner for the nonlinear conjugate gradient (NCG) algorithm. Alternatively, we can view the NCG algorithm as a way to accelerate the ALS iterates. To investigate the performance of our resulting preconditioned nonlinear conjugate gradient (PC-NCG) algorithm we perform numerical tests using our algorithm to factorize a number of different tensors.

## Nonlinear Eigenvalue Problems

### A Padé Approximate Linearization Technique for Solving the Quadratic Eigenvalue Problem with Low-Rank Damping

Xin Huang, Ding Lu, Yangfeng Su, Zhaojun Bai

The low-rank damping term appears commonly in the quadratic eigenvalue problem arising from real physical simulations. To exploit this low-rank property, we propose an Approximate Linearization technique via padeapproximants, PAL in short. One of advantages of the PAL technique is that the dimension of the resulting linear eigenvalue problem is only $n + rm$, which is generally substantial smaller than the dimension $2n$ of the linear eigenvalue problem derived by an exact linearization scheme, where $n$ is the dimension of the original QEP, $r$ and $m$ are the rank of the damping matrix and the order of padeapproximant, respectively. In addition, we propose a scaling strategy aiming at the minimization of the backward error of the PAL approach.

### Distance problems for Hermitian matrix polynomials

Shreemayee Bora, Ravi Srivastava

Hermitian matrix polynomials with hyperbolic, quasi-hyperbolic or definite structure arise in a number of applications in science and engineering. A distinguishing feature of such polynomials is that their eigenvalues are real with the associated sign characteristic being either positive or negative. We propose a definition of the sign characteristic of a real eigenvalue of a Hermitian polynomial which is based on the homogeneous form of the polynomial and use it to analyze the distance from a given member of each of these classes to a nearest member outside the class with respect to a specified norm. The analysis is based on Hermitian $\epsilon$-pseudospectra and results in algorithms based on the bisection method to compute these distances. The main computational work in each step of bisection involves finding either the largest eigenvalue/eigenvalues of a positive definite matrix or the smallest eigenvalue/eigenvalues of a negative definite matrix along with corresponding eigenvectors and the algorithms work for all the above mentioned structures except for certain classes of definite polynomials.

### Spectral equivalence of Matrix Polynomials, the Index Sum Theorem and consequences

Fernando De Teran, Froilan Dopico, Steve Mackey

In this talk we introduce the *spectral equivalence* relation on matrix polynomials (both square and rectangular). We show that the invariants of this equivalence relation are: (a) The dimension of the left and right null spaces, (b) the finite elementary divisors, and (c) the infinite elementary divisors. We also compare this relation with other equivalence relations in the literature [1,2]. We analyze what happens with the remaining part of the "eigenstructure", namely, the left and right minimal indices, of two spectrally equivalent matrix polynomials. Particular cases of spectrally equivalent polynomials of a given one are *linearizations* (polynomials of degree one) and, more in general, *$\ell$-ifications* (polynomials of degree $\ell$). We analyze the role played by the degree in the spectral equivalence. This leads us to the Index Sum Theorem, an elementary relation between the sum of the finite and infinite structural indices (degrees of elementary divisors), the sum of all minimal indices, the degree, and the rank of any matrix polynomial. Though elementary in both the statement and proof, the Index Sum Theorem has several powerful consequences. We show some of them, mostly related to structured $\ell$-ifications, in the last part of the talk.


[1] N. Karampetakis, S. Vologiannidis. Infinite elementary divisor structure-preserving transformations for polynomial matrices. *Int. J. Appl. Math. Comput. Sci.*, 13 (2003) 493–503.

[2] A. C. Pugh, A. K. Shelton. On a new definition of strict system equivalence. *Int. J. Control*, 27 (1978) 657–672.


### Block iterations for eigenvector nonlinearities

Elias Jarlebring

Let $A(V) \in \mathbb{R}^{n\times n}$ be a symmetric matrix depending on $V \in \mathbb{R}^{n\times k}$ which is a basis of a vector space and such that $A(V)$ is independent of the choice of basis of the vector space. We here consider the problem of computing $V$ such that $(\Lambda, V)$ is an invariant pair of the matrix $A(V)$, i.e., $A(V)V = V\Lambda$. We present a block algorithm for this problem, where every step involves solving one or several linear systems of equations. The algorithm is a generalization of (shift-and-invert) simultaneous iteration for the standard eigenvalue problem and we show that the generalization inherits many of its properties. The algorithm is illustrated with the application to a model problem in quantum chemistry.

### A Filtration Perspective on Minimal Indices

D. Steven Mackey

In addition to eigenvalues, singular matrix polynomials also possess scalar invariants called "minimal indices" that are significant in a number of application areas, such as control theory and linear systems theory. In this talk we discuss a new approach to the definition of these quantities based on the notion of a filtration of a vector space. The goal of this filtration formulation is to provide new tools for working effectively with minimal indices, as well as to gain deeper insight into their fundamental nature. Several illustrations of the use of these tools will be given.

### An overview of Krylov methods for nonlinear eigenvalue problems

Karl Meerbergen, Roel Van Beeumen, Wim Michiels

This talk is about the solution of a class of non-linear eigenvalue problems. Krylov and Rational Krylov methods are known to be efficient and reliable for the solution of linear and polynomial eigenvalue problems. In earlier work, we suggested to approximate the nonlinear matrix function by interpolating polynomials which led to the Taylor-Arnoldi, infinite Arnoldi and Newton-Hermite Rational Krylov methods. These methods are flexible in that the number of iterations and the interpolation points can be chosen during the execution of the algorithm. All these methods have in common that they interpolate, i.e., they match moments of a related dynamical system with nonlinear dependence on the eigenvalue parameter. The difference lies in a different innerproduct that is used for an infinite dimensional problem. Numerical experiments showed that the impact of a specific innerproduct is hard to understand. This leads to the conclusion that we better project the nonlinear eigenvalue problem on the subspace of the moment vectors, leading to a small scale nonlinear eigenvalue problem. The disadvantage is that we again have to solve a nonlinear eigenvalue problem with small matrices this time. We will again use interpolating polynomials within a rational Krylov framework for the small scale problem. We then give an overview of different possibilities for restarting interpolation based Krylov methods, which allows for computing invariant pairs of nonlinear eigenvalue problems. We will also formulate open questions, both small and large scale problems.

### Skew-symmetry - a powerful structure for matrix polynomials

Steve Mackey, Niloufer Mackey, Christian Mehl, Volker Mehrmann

We consider skew-symmetric matrix polynomials, i.e., matrix polynomials, where all coefficient matrices are skew-symmetric.

In the talk, we develop a skew-symmetric Smith form for skew-symmetric matrix polynomials under structure preserving unimodular equivalence transformations. From this point of view, the structure of being skew-symmetric proves to be powerful, because analogous results for other structured matrix polynomials are not available.

We will apply the so-called skew-Smith form to deduce a result on the existence of skew-symmetric strong linearizations of regular skew-symmetric matrix polynomials. This is in contrast to other important structures, like T-alternating and T-palindromic polynomials, where restrictions in the elementary divisor structure of matrix polynomials and matrix pencils have been observed leading to the effect that structured strong linearizations of such structured matrix polynomials need not always exist.

Further applications of the skew-Smith form presented in the talk include the solution of a problem on the existence of symmetric factorizations of skew-symmetric rational functions.

### Vector spaces of linearizations for matrix polynomials: a bivariate polynomial approach

Alex Townsend, Vanni Noferini, Yuji Nakatsukasa

We revisit the important paper [D. S. Mackey, N. Mackey, C. Mehl and V. Mehrmann, *SIAM J. Matrix Anal. Appl.*, 28 (2005), pp. 971–1004] and by viewing matrices as coefficients for bivariate polynomials we provide concise proofs for key properties of linearizations of matrix polynomials. We also show that every pencil in the double ansatz space is intrinsically connected to a Bézout matrix, which can be used to prove the eigenvalue exclusion theorem. Furthermore, we exploit this connection in order to obtain a new result on the Kronecker canonical form of $\mathbb{DL}(P, v)$ in the case where this pencil is not a linearization.

In addition, our exposition allows for any degree-graded basis, the monomials being a special case.

### Duality of matrix pencils, singular pencils and linearizations

Vanni Noferini, Federico Poloni

We consider a new relation among matrix pencils, duality: two pencils $xL_1 + L_0$ and $xR_1 + R_0$ are said to be duals if $L_1R_0 = R_1L_0$, plus a rank condition. Duality transforms Kronecker blocks in a predictable way, and can be used as a theoretical tool to derive results in different areas of the theory of matrix pencils and linearizations. We discuss Wong chains, a generalization of eigenvalues and Jordan chains to singular pencils, and how they change under duality. We use these tools to derive a result on the possible minimal indices of Hamiltonian and symplectic pencils, to study vector spaces of linearizations as in [Mackey$^2$, Mehrmann, Mehl 2006], and to revisit the proof that Fiedler pencils associated to singular matrix polynomials are linearizations [De Terán, Dopico, Mackey 2009].

### Exploiting low rank of damping matrices using the Ehrlich-Aberth method

Leo Taslaman

We consider quadratic matrix polynomials $Q(\lambda) = M\lambda^2 + D\lambda + K$ corresponding to vibrating systems with low rank damping matrix $D$. Our goal is to exploit the low rank property of $D$ to compute all eigenvalues of $Q(\lambda)$ more efficiently than conventional methods. To this end, we use the Ehrlich-Aberth method recently investigated by Bini and Noferini for computing the eigenvalues of matrix polynomials. For general matrix polynomials, the Ehrlich-Aberth method computes eigenvalues with good accuracy but is unfortunately relatively slow when the size of the matrix polynomial is large and the degree is low. We revise the algorithm and exploit the special structure of our systems to push down the bulk sub-computation from cubic to linear time (in matrix size) and obtain an algorithm that is both fast and accurate.

### A Review of Nonlinear Eigenvalue Problems

Francoise Tisseur

In its most general form, the nonlinear eigenvalue problem is to find scalars $\lambda$ (eigenvalues) and nonzero vectors $x$ and $y$ (right and left eigenvectors) satisfying $N(\lambda)x = 0$ and $y^*N(\lambda) = 0$, where $N\colon \Omega \to \mathbb{C}$ is an analytic function on an open set $\Omega \subseteq \mathbb{C}$. In practice, the matrix elements are most often polynomial, rational or exponential functions of $\lambda$ or a combination of these. These problems underpin many areas of computational science and engineering. They can be difficult to solve due to large problem size, ill conditioning (which means that the problem is very sensitive to perturbations and hence hard to solve accurately), or simply a lack of good numerical methods. My aim is to review the recent research directions in this area.

### The Inverse Symmetric Quadratic Eigenvalue Problem

Ion Zaballa, Peter Lancaster

The detailed spectral structure of symmetric quadratic matrix polynomials has been developed recently. Their canonical forms are shown to be useful to provide a detailed analysis of inverse problems of the form: construct the coefficients of a symmetric quadratic matrix polynomial from the spectral data including the classical eigenvalue/eigenvector data and sign characteristics for the real eigenvalues. An orthogonality condition dependent on these signs plays a vital role in this construction. Special attention is paid to the cases when the leading and trailing coefficients are prescribed to be positive definite. A precise knowledge of the admissible sign characteristics of such matrix polynomials is revealed to be important.

## Randomized Matrix Algorithms

### A Randomized Asynchronous Linear Solver with Provable Convergence Rate

Haim Avron, Alex Druinsky, Anshul Gupta

Asynchronous methods for solving systems of linear equations have been researched since Chazan and Miranker published their pioneering paper on chaotic relaxation in 1969. The underlying idea of asynchronous methods is to avoid processor idle time by allowing the processors to continue to work and make progress even if not all progress made by other processors has been communicated to them.

Historically, work on asynchronous methods for solving linear equations focused on proving convergence in the limit. How the rate of convergence compares to the rate of convergence of the synchronous counterparts, and how it scales when the number of processors increase, was seldom studied and is still not well understood. Furthermore, the applicability of these methods was limited to restricted classes of matrices (e.g., diagonally dominant matrices).

We propose a shared-memory asynchronous method for general symmetric positive definite matrices. We rigorously analyze the convergence rate and prove that it is linear and close to that of our method's synchronous counterpart as long as not too many processors are used (relative to the size and sparsity of the matrix). A key component is randomization, which allows the processors to make guaranteed progress without introducing synchronization. Our analysis shows a convergence rate that is linear in the condition number of the matrix, and depends on the number of processors and the degree to which the matrix is sparse.

### Randomized low-rank approximations, in theory and practice

Alex Gittens

Recent research has sharpened our understanding of certain classes of low-rank approximations formed using randomization. In this talk, we consider two classes of such approximants: those of general matrices formed using subsampled fast unitary transformations, and Nystrom extensions of SPSD matrices. We see that despite the fact that the available bounds are often optimal or near-optimal in the worst case, in practice these methods are much better behaved than the bounds predict. We consider possible explanations for this gap, and potential ways to incorporate assumptions on our datasets to further increase the predictive value of the theory.

### Accuracy of Leverage Score Estimation

Ilse Ipsen, John Holodnak

Leverage scores were introduced in 1978 by Hoaglin and Welsch for outlier detection in statistical regression analysis. Starting about ten years ago, Mahoney et al. pioneered the use of leverage scores for importance sampling in randomized algorithms for matrix computations. Subsequently they developed sketching-based algorithms for estimating leverage scores, in order to reduce the operation count of $O(mn^2)$ for a $m \times n$ full-column rank matrix. We present tighter error bounds for the leverage approximations produced by these sketching-based algorithms.

### Implementing Randomized Matrix Algorithms in Parallel and Distributed Environments

Michael Mahoney

Motivated by problems in large-scale data analysis, randomized algorithms for matrix problems such as regression and low-rank matrix approximation have been the focus of a great deal of attention in recent years. These algorithms exploit novel random sampling and random projection methods; and implementations of these algorithms have already proven superior to traditional state-of-the-art algorithms, as implemented in Lapack and high-quality scientific computing software, for moderately-large problems stored in RAM on a single machine. Here, we describe the extension of these methods to computing high-precision solutions in parallel and distributed environments that are more common in very large-scale data analysis applications.

In particular, we consider both the Least Squares Approximation problem and the Least Absolute Deviation problem, and we develop and implement randomized algorithms that take advantage of modern computer architectures in order to achieve improved communication profiles. Our iterative least-squares solver, LSRN, is competitive with state-of-the-art implementations on moderately-large problems; and, when coupled with the Chebyshev semi-iterative method, scales well for solving large problems on clusters that have high communication costs such as on an Amazon Elastic Compute Cloud cluster. Our iterative least-absolute-deviations solver is based on fast ellipsoidal rounding, random sampling, and interior-point cutting-plane methods; and we demonstrate significant improvements over traditional algorithms on MapReduce. In addition, this algorithm can also be extended to solve more general convex problems on MapReduce.

## Sign Pattern Matrices

### Sign Vectors and Duality in Rational Realization of the Minimum Rank

Marina Arav, Frank Hall, Zhongshan Li, Hein van der Holst

The minimum rank of a sign pattern matrix $A$ is the minimum of the ranks of the real matrices whose entries have signs equal to the corresponding entries of $A$. We prove that rational realization for any $m \times n$ sign pattern matrix $A$ with minimum rank $n-2$ is always possible. This is done using a new approach involving sign vectors and duality. Furthermore, we show that for each integer $n \geqslant 12$, there exists a nonnegative integer $m$ such that there exists an $n \times m$ sign pattern matrix with minimum rank $n-3$ for which rational realization is not possible.

### Sign patterns that require eventual exponential nonnegativity

Craig Erickson

A real square matrix $A$ is eventually exponentially nonnegative if there exists a positive real number $t_0$ such that for all $t \geqslant t_0$, $e^{tA}$ is an entrywise nonnegative matrix where $e^{tA} = \sum_{k=0}^{\infty} \frac{t^k A^k}{k!}$. A sign pattern $\mathcal{A}$ is a matrix having entries in $\{+, -, 0\}$ and its qualitative class is the set of all real matrices $A$ for which $\operatorname{sgn}(A) = \mathcal{A}$. A sign pattern $\mathcal{A}$ requires eventual exponential nonnegativity if every matrix in the qualitative class of $\mathcal{A}$ is eventually exponentially nonnegative. In this talk, we discuss the structure of sign patterns that require eventual exponential nonnegativity.

### Sign patterns with minimum rank 3

Wei Gao, Yubin Gao, Fei Gong, Guangming Jing, Zhongshan Li

A *sign pattern (matrix)* is a matrix whose entries are from the set $\{+, -, 0\}$. The *minimum rank* (respectively, *rational minimum rank*) of a sign pattern matrix $\mathcal{A}$ is the minimum of the ranks of the real (respectively, rational) matrices whose entries have signs equal to the corresponding entries of $\mathcal{A}$. It is known that for each sign pattern $\mathcal{A} = [\mathbf{a}_{ij}]$ with minimum rank 2, there is a rank 2 integer matrix $A = [b_{ij}]$ with $\operatorname{sgn}(b_{ij}) = \mathbf{a}_{ij}$. A sign pattern $\mathcal{A}$ is said to be *condensed* if $\mathcal{A}$ has no zero row or column and no two rows or columns are identical or negatives of each other. In this talk, we establish a new direct connection between condensed $m \times n$ sign patterns with minimum rank 3 and $m$ point-$n$ line configurations in the plane, and we use this connection to construct the smallest known sign pattern whose minimum rank is 3 but whose rational minimum rank is greater than 3. We also obtain other results in this direction.

### Sign patterns that require or allow generalizations of nonnegativity

Leslie Hogben

A real square matrix $A$ is eventually positive (nonnegative) if $A^k$ is an entrywise positive (nonnegative) matrix for all sufficiently large $k$. A matrix $A$ is power-positive if some positive integer power of $A$ is entrywise positive. A matrix $A$ is eventually exponentially positive if there exists some $t_0 \geqslant 0$ such that for all $t \geqslant t_0$, $e^{tA} = \sum_{k=0}^{\infty} \frac{t^k A^k}{k!} > 0$.

A sign pattern $\mathcal{A}$ requires a property $P$ if every real matrix $A$ having sign pattern $\mathcal{A}$ has property $P$, and allows a property $P$ if there is a real matrix $A$ having sign pattern $\mathcal{A}$ that has property $P$. This talk will survey recent results about sign patterns that require or allow these various eventual properties that generalize nonnegativity.

### Sign Patterns with minimum rank 2 and upper bounds on minimum ranks

Zhongshan Li, Wei Gao, Yubin Gao, Fei Gong, Marina Arav

A sign pattern (matrix) is a matrix whose entries are from the set $\{+, -, 0\}$. The *minimum rank* (resp., rational minimum rank) of a sign pattern matrix $A$ is the minimum of the ranks of the real (resp., rational) matrices whose entries have signs equal to the corresponding entries of $A$. The notion of a condensed sign pattern is introduced. A new, insightful proof of the rational realizability of the minimum rank of a sign pattern with minimum rank 2 is obtained. Several characterizations of sign patterns with minimum rank 2 are established, along with linear upper bounds for the absolute values of an integer matrix achieving the minimum rank 2. A known upper bound for the minimum rank of a $(+, -)$ sign pattern in terms of the maximum number of sign changes in the rows of the sign pattern is substantially extended to obtain upper bounds for the rational minimum ranks of general sign pattern matrices. The new concept of the number of polynomial sign changes of a sign vector is crucial for this extension. Another known upper bound for the minimum rank of a $(+,-)$ sign pattern in terms of the smallest number of sign changes in the rows of the sign pattern is also extended to all sign patterns using the notion of the number of strict sign changes. Some examples and open problems are also presented.

**Properties of Skew-Symmetric Adjacency Matrices**

Judi McDonald

Given an graph on $n$ vertices, a skew-symmetric adjacency matrix can be formed by choosing $a_{ij} = \pm 1$, whenever there is an edge from vertex $i$ to vertex $j$, in such a way that $a_{ij}a_{ji} = -1$, and setting $a_{ij} = 0$ otherwise. Thus there is a collection of skew-symmetric matrices that can be associated with each graph. In the talk, characteristics of this collection will be discussed.

**Using the Nilpotent-Jacobian Method Over Various Fields and Counterexamples to the Superpattern Conjecture.**

Timothy Melvin

The Nilpotent-Jacobian Method is a powerful tool used to determine if a pattern (and its superpatterns) is spectraly arbitrary over $\mathbb{R}$. We will explore what information can be gleaned from this method when we consider patterns over various fields including finite fields, $\mathbb{Q}$, and algebraic extensions of $\mathbb{Q}$. We will also provide a number of counterexamples to the Super Pattern Conjecture over the finite field $\mathbb{F}_3$.

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**Structure and Randomization in Matrix Computations**

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**Accelerating linear system solutions using randomization**

Marc Baboulin, Jack Dongarra

Recent years have seen an increase in peak "local" speed through parallelism in terms of multicore processors and GPU accelerators. At the same time, the cost of communication between memory hierarchies and/or between processors have become a major bottleneck for most linear algebra algorithms. In this presentation we describe a randomized algorithm that accelerates the factorization of general or symmetric indefinite systems on multicore or hybrid multicore+GPU systems. Randomization prevents the communication overhead due to pivoting, is computationally inexpensive and requires very little storage. The resulting solvers outperform existing routines while providing us with a satisfying accuracy.

**The bisection method and condition numbers for quasiseparable of order one matrices**

Yuli Eidelman, Iulian Haimovici

We discuss the fast bisection method to compute all or selected eigenvalues of quasiseparable of order one matrices. The method is based essentially on fast evaluation formulas for characteristic polinomials of principal leading submatrices of quasiseparable of order one matrices obtained in [1]. In the implementation of the method the problem of fast computing of conditional numbers for quasiseparable of order one matrices arises in a natural way. The results of numerical tests are discussed.

[1] Y. Eidelman I. Gohberg and V. Olshevsky, Eigenstructure of order-one-quasiseparable matrices. Three-term and two-term tecurrence relations, LAA, 405 (2005), 1-40.

**The unitary eigenvalue problem**

Luca Gemignani

The talk is concerned with the computation of a condensed representation of a unitary matrix. A block Lanczos procedure for the reduction of a unitary matrix into a CMV-like form is investigated. The reduction is used as a pre-processing phase for eigenvalue computation by means of the QR method. Numerical results and some potential applications to polynomial root-finding will be discussed. This is a joint work with R. Bevilacqua and G. Del Corso.

**Randomly sampling from orthonormal matrices: Coherence and Leverage Scores**

Ilse Ipsen, Thomas Wentworth

We consider three strategies for sampling rows from matrices Q with orthonormal columns: Exactly(c) without replacement, Exactly(c) with replacement, and Bernoulli sampling.

We present different probabilistic bounds for the condition numbers of the sampled matrices, and express them in terms of the coherence or the leverage scores of Q. We also present lower bounds for the number of sampled rows to attain a desired condition number. Numerical experiments confirm the accuracy of the bounds, even for small matrix dimensions.

**Inverse eigenvalue problems linked to rational Arnoldi, and rational (non)symmetric Lanczos**

Thomas Mach, Marc Van Barel, Raf Vandebril

In this talk two inverse eigenvalue problems are discussed. First, given the eigenvalues and a weight vector a (extended) Hessenberg matrix is computed. This matrix represents the recurrences linked to a (rational) Arnoldi inverse problem. It is well known that the matrix capturing the recurrence coefficients is of Hessenberg form in the standard Arnoldi case. Considering, however, rational functions, admitting finite as well as infinite poles we will prove that the recurrence matrix is still of a highly structured form – the extended Hessenberg form. An efficient memory representation is presented for the Hermitian case and used to reconstruct the matrix capturing the recurrence coefficients.

In the second inverse problem, the above setting is generalized to the biorthogonal case. Instead of unitary similarity transformations, we drop the unitarity. Given the eigenvalues and the two first columns of the matrices determining the similarity, we want to retrieve the matrix of recurrences, as well as the matrices governing the transformation.

**Decoupling randomness and vector space structure leads to high-quality randomized matrix algorithms**

Michael Mahoney

Recent years have witnessed the increasingly sophisticated use of randomness as a computational resource in the development of improved algorithms for fundamental linear algebra problems such as least-squares approximation, low-rank matrix approximation, etc. The best algorithms (in worst-case theory, in high-quality numerical implementation in RAM, in large-scale parallel and distributed environments, etc.) are those that decouple, either explicitly in the algorithm or implicitly within the analysis, the Euclidean vector space structure (e.g., the leverage score structure, as well as other more subtle variants) from the algorithmic application of the randomness. For example, doing so leads to much better a priori bounds on solution quality and running time, more flexible ways to parameterize algorithms to make them appropriate for particular applications, as well as structural conditions that can be checked a posteriori. Although this perspective is quite different than more traditional approaches to linear algebra, the careful use of random sampling and random projection has already led to the development of qualitatively different and improved algorithms, both for matrices arising in more traditional scientific computing applications, as well as for matrices arising in large-scale machine learning and data analytics applications. This approach will be described, and several examples of it will be given.

**Transformations of Matrix Structures Work Again**

Victor Pan

In 1989 we proposed to employ Vandermonde and Hankel multipliers to transform into each other the matrix structures of Toeplitz, Hankel, Vandermonde and Cauchy types as a means of extending any successful algorithm for the inversion of matrices having one of these structures to inverting the matrices with the structures of the three other types. Surprising power of this approach has been demonstrated in a number of works, which culminated in ingeneous numerically stable algorithms that approximated the solution of a nonsingular Toeplitz linear system in nearly linear (versus previuosly cubic) arithmetic time. We first revisit this powerful method, covering it comprehensively, and then specialize it to yield a similar acceleration of the known algorithms for computations with matrices having structures of Vandermonde or Cauchy types. In particular we arrive at numerically stable approximate multipoint polynomial evaluation and interpolation in nearly linear time, by using $O(bn\log^h n)$ flops where $h = 1$ for evaluation, $h = 2$ for interpolation, and $2^{-b}$ is the relative norm of the approximation errors.

**On the Power of Multiplication by Random Matrices**

Victor Pan

A random matrix is likely to be well conditioned, and motivated by this well known property we employ random matrix multipliers to advance some fundamental matrix computations. This includes numerical stabilization of Gaussian elimination with no pivoting as well as block Gaussian elimination, approximation of the leading and trailing singular spaces of an ill conditioned matrix, associated with its largest and smallest singular values, respectively, and approximation of this matrix by low-rank matrices, with extensions to the computation and certification of the numerical rank of a matrix and its 2-by-2 block triangular factorization, where its rank exceeds its numerical rank. We prove the efficiency of the proposed techniques where we employ Gaussian random multipliers, but our extensive tests have consistently produced the same outcome where instead we used sparse and structured random multipliers, defined by much fewer random parameters than the number of the input entries.

**Multilevel low-rank approximation preconditioners**

Yousef Saad, Ruipeng Li

A new class of methods based on low-rank approximations which has some appealing features will be introduced. The methods handle indefiniteness quite well and are more amenable to SIMD computations, which makes them attractive for GPUs. The method is easily defined for Symmetric Positive Definite model problems arising from Finite Difference discretizations of PDEs. We will show how to extend to general situations using domain decomposition concepts.

**Structured matrix polynomials and their sign characteristic**

Maha Al-Ammari, Steve Mackey, Yuji Nakatsukasa, Francoise Tisseur

Matrix polynomials $P(\lambda) = \lambda^m A_m + \cdots + \lambda A_1 + A_0$ with Hermitian or symmetric matrix coefficients $A_i$ or with coefficients which alternate between Hermitian and skew-Hermitian matrices, or even with coefficient matrices appearing in a palindromic way commonly arise in applications.

Standard and Jordan triples $(X, J, Y)$ play a central role in the theory of matrix polynomials with nonsingular leading coefficient. They extend to matrix polynomials the notion of Jordan pairs $(X, J)$ for a single matrix $A$, where $A = X^{-1}JX$. Indeed, each matrix coefficient $A_i$ of $P(\lambda)$ can be expressed in terms of $X, J$ and $Y$. We show that standard triples of structured $P(\lambda)$ have extra properties. In particular there is a nonsingular matrix $M$ such that $MY = v_{\mathcal{S}}(J)X^*$, where $v_{\mathcal{S}}(J)$ depends on the structure $\mathcal{S}$ of $P(\lambda)$ and $J$ is $M$-selfadjoint when $\mathcal{S} \in \{$Hermitian, skew-Hermitian$\}$, $M$-skew-adjoint when $\mathcal{S} \in \{*$-even, $*$-odd$\}$ and $M$-unitary when $\mathcal{S} \in \{$palindromic, antipalindromic$\}$.

The property of $J$ implies that its eigenvalues and therefore those of $P(\lambda)$ occur in pairs $(\lambda, f(\lambda))$ when $\lambda \neq f(\lambda)$, where

$$f(\lambda) = \begin{cases} \overline{\lambda} & \text{for } M\text{-self-adjoint } J, \\ -\overline{\lambda} & \text{for } M\text{-skew-adjoint } J, \\ 1/\overline{\lambda} & \text{for } M\text{-unitary } J. \end{cases}$$

The eigenvalues for which $\lambda = f(\lambda)$, that is, those that are not paired, have a sign $+1$ or $-1$ attached to them forming the *sign characteristic of* $P(\lambda)$. We define the sign characteristic of $P(\lambda)$ as that of the pair $(J, M)$, show how to compute it and study its properties. We discuss applications of the sign characteristic in particular in control systems, in the solution of structured inverse polynomial eigenvalue problems and in the characterization of special structured matrix polynomials such as overdamped quadratics, hyperbolic and quasidefinite matrix polynomials.

**On solving indefinite least squares-type problems via anti-triangular factorization**

Nicola Mastronardi, Paul Van Dooren

The indefinite least squares problem and the equality constrained indefinite least squares problem are modifications of the least squares problem and the equality constrained least squares problem, respectively, involving the minimization of a certain type of indefinite quadratic form. Such problems arise in the solution of Total Least Squares problems, in parameter estimation and in $H_\infty$ smoothing. Algorithms for computing the numerical solution of indefinite least squares and indefinite least squares with equality constraint are described by Bojanczyk et al. and Chandrasekharan et al.

The indefinite least squares problem and the equality constrained indefinite least squares problem can be expressed in an equivalent fashion as augmented square linear systems. Exploiting the particular structures of the coefficient matrices of such systems, new algorithms for computing the solution of such problems are proposed relying on the anti-triangular factorization of the coefficient matrix (by Mastronardi et al.). Some results on their stability are shown together with some numerical examples.

**Randomized and matrix-free structured sparse direct solvers**

Jianlin Xia

We show how randomization and rank structures can be used in the direct solution of large sparse linear systems with nearly O(n) complexity and storage. Randomized sampling and related adaptive strategies help significantly improve both the efficiency and flexibility of structured solution techniques. We also demonstrate how these can be extended to the development of matrix-free direct solvers based on matrix-vector products only. This is especially interesting for problems with few varying parameters (e.g., frequency or shift). Certain analysis of the errors and stability will be given. Among many of the applications are the solutions of large discretized PDEs and some eigenvalue problems.

We also consider the issues of using the techniques in theses contexts:

1. Finding selected as well as arbitrary entries of the inverse of large sparse matrices, and the cost is O(n) for O(n) entries when the matrices arise from the discretization of certain PDEs in both 2D and 3D.

2. The effectiveness of using these methods as incomplete structured preconditioners or even matrix-free preconditioners.

Part of the work is joint with Yuanzhe Xi.

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**Structured Matrix Functions and their Applications (Dedicated to Leonia Lerer on the occasion of his 70th birthday)**

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**Structured singular-value analysis and structured Stein inequalities**

Joseph Ball

For $A$ an $n \times n$ matrix, it is well known that $A$ is stable (i.e., $I - zA$ is invertible fora ll $z$ in the closed unit disk $\overline{\mathbf{D}}$) if and only if there exists a positive definite solution $X \succ 0$ to the strict Stein inequality $X - A^*XA \succ 0$. Let us now specify a block decomposition $A = \left[\begin{smallmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{smallmatrix}\right]$ and say that $A$ is *structured stable* if $\left[\begin{smallmatrix} I_{n_1} & 0 \\ 0 & I_{n_2} \end{smallmatrix}\right] - \left[\begin{smallmatrix} z_1 I_{n_1} & 0 \\ 0 & z_2 I_{n_2} \end{smallmatrix}\right] \left[\begin{smallmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{smallmatrix}\right]$ is invertible for all $(z_1, z_2)$ in the closed bidisk $\overline{\mathbb{D}} \times \overline{\mathbb{D}}$. It is known that structured stability of $A$ is implied by but in general does not imply the existence of a positive definite structured solution $X = \left[\begin{smallmatrix} X_1 & 0 \\ 0 & X_2 \end{smallmatrix}\right]$ to the Stein inequality $X - A^*XA \succ 0$. However, if one enhances the structure by replacing $A$ by $A \otimes I_{\ell^2}$, one does get an equivalence of robust stability with existence of a solution with a structured LMI, i.e., the existence of a structured positive definite solution $X$ to the Stein inequality $X - A^*XA \succ 0$ is equivalent to the enhanced notion of robust stability: $I_{\ell^2_n} - \left[\begin{smallmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{smallmatrix}\right] \left(A \otimes I_{\ell^2_n}\right)$ invertible for all $(\Delta_1, \Delta_2)$ in the noncommutative closed bidisk $\overline{\mathcal{B}}\mathcal{L}(\ell^2_{n_1}) \times \overline{\mathcal{B}}\mathcal{L}(\ell^2_{n_2})$. These ideas are important in stability and performance analysis and synthesis for multidimensional systems as well as for standard input/state/output linear systems carrying a linear-fractional-transformation model of structured uncertainty (structured singular value or $\mu$-analysis—see [1]). We present here a unified framework for formulating and verifying the result which incorporates non-square block structure as well as arbitrary block-multiplicity. This talk discusses joint work with Gilbert Groenewald and Sanne ter Horst of North-West University, Potchefstroom, South Africa.

**Zero sums of idempotents and Banach algebras failing to be spectrally regular**

Harm Bart

A basic result from complex function theory states that the contour integral of the logarithmic derivative of a scalar analytic function can only vanish when the function has no zeros inside the contour. The question dealt with here is: does this result generalize to the vector-valued case? We suppose the function takes values in a Banach algebra. In case of a positive answer, the Banach algebra is called *spectrally regular*. There are many of such algebras. In this presentation, the focus will be on negative answers so on Banach algebras failing to be spectrally regular. These have been identified via the construction of non-trivial zero sums of a finite number of idempotents. It is intriguing that we need only five idempotents in all known examples. The idempotent constructions relate to deep problems concerning the geometry of Banach spaces and general topology.

The talk is a report on work done jointly with Torsten Ehrhardt (Santa Cruz) and Bernd Silbermann (Chemnitz).

**An overview of Leonia Lerer's work**

Marinus Kaashoek

In this talk I will present on overview of some of Leonia Lerer's many contributions to matrix and operator theory. The emphasis will be on Szego-Krein orthogonal matrix polynomials and Krein orthogonal entire matrix functions.

**Trace formulas for truncated block Toeplitz operators**

David Kimsey

The strong Szegő limit theorem may be formulated in terms of operators of the form

$$(P_N \mathfrak{G} P_N)^n - P_N \mathfrak{G}^n P_N, \quad \text{for } n = 1, 2, \dots,$$

where $\mathfrak{G}$ denotes the operator of multiplication by a suitably restricted $d \times d$ mvf (matrix-valued function) acting on the space of $d \times 1$ vvf's (vector-valued functions) $f$ that meet the constraint $\int_0^{2\pi} f(e^{i\theta})^* \Delta(e^{i\theta}) f(e^{i\theta}) d\theta < \infty$ with $\Delta(e^{i\theta}) = I_d$ and $P_N$ denotes the orthogonal projection onto the space of vvf's $f$ with Fourier series $f(e^{i\theta}) = \sum_{k=-N}^{N} e^{ik\theta} f_k$ that are subject to the same summability constraint. We shall study these operators for a class of positive definite mvf's $\Delta$, which admit the factorization $\Delta(e^{i\theta}) = Q(e^{i\theta})^* Q(e^{i\theta}) = R(e^{i\theta}) R(e^{i\theta})^*$, where $Q^{\pm 1}$ and $R^{\pm 1}$ belong to a certain algebra. We show that the limit

$$\kappa_n(G) = \lim_{N \uparrow \infty} \text{trace}\{(P_N \mathfrak{G} P_N)^n - P_N \mathfrak{G}^n P_N\}$$

exists and by may evaluated using $\Delta(e^{i\theta}) = I_d$, when $GQ = QG$ and $R^* G = R^* G$. Moreover, when $G$ possesses a special factorization,

$$\kappa_n(G) = -\frac{n}{d} \sum_{k=1}^{n-1} \sum_{j=1}^{\infty} j \left( \frac{\text{trace}([G^k]_j)}{k} \right) \left( \frac{\text{trace}([G^{n-k}]_{-j})}{n-k} \right).$$

We will also discuss analogous results for operators which are connected to a continuous analog of the strong Szegő limit theorem.

This talk is based on joint work with Harry Dym.

**Norm asymptotics for a special class of Toeplitz generated matrices with perturbations**

Hermann Rabe

Matrices of the form $X_n = T_n + c_n (T_n^{-1})^*$, where $T_n$ is an invertible banded Toeplitz matrix, and $c_n$ a sequence of positive real numbers are considered. The norms of their inverses are described asymptotically as their $(n \times n)$ sizes increase. Certain finite rank perturbations of these matrices are shown to have no effect on their asymptotic behaviour.

**One sided invertibility, corona problems, and Toeplitz operators**

Leiba Rodman

We study Fredholm properties of Toeplitz operators with matrix symbols acting on Hardy space of the upper halfplane, in relation to the Fredholm properties of Toeplitz operators with scalar determinantal symbols. The study is based on analysis of one-sided invertibility in the abstract framework of unital commutative ring, and of Riemann - Hilbert problem in connection to pairs of matrix functions that satisfy the corona condition. The talk is based on joint work with M.C. Camara and I.M. Spitkovsky.

**Discrete Dirac systems and structured matrices**

Alexander Sakhnovich

We show that discrete Dirac system is equivalent to Szegö recurrence. Discrete Dirac system is also of independent interest. We present Weyl theory for discrete Dirac systems with rectangular matrix "potentials". Block structured matrices are essential for solution of the corresponding inverse problems. The results were obtained jointly with B. Fritzsche, B. Kirstein and I. Roitberg.

### On almost normal matrices

Tyler Moran , Ilya Spitkovsky

Let an $n$-by-$n$ matrix $A$ be almost normal in a sense that it has $n-1$ orthogonal eigenvectors. The properties of its numerical range $W(A)$ and Aluthge transform $\Delta$ are explored. In particular, it is proven that for unitarily irreducible almost normal $A$, $W(A)$ cannot have flat portions on the boundary and $\Delta(A)$ is not normal (the latter, under the additional conditions that $n > 2$ and $A$ is invertible). In passing, the unitary irreducibility criterion for $A$, almost normality criterion for $A^*$, and the rank formula for the self-commutator $A^*A - AA^*$ are established.

### Toeplitz plus Hankel (and Alternating Hankel)

Gilbert Strang

A symmetric tridiagonal Toeplitz matrix $T$ has the strong $TH$ property:

All functions of $T$ can be written as a Toeplitz matrix plus a Hankel matrix.

We add two families of matrices to this class:

1. The entries in the four corners of $T$ can be arbitrary (with symmetry)
2. $T$ can be completed to have a tridiagonal inverse (Kac-Murdock-Szego matrices have entries $cR^{|i-j|}$ with rank 1 above the diagonal)

The antisymmetric centered difference matrix $D$ has a parallel property except that its antidiagonals alternate in sign (alternating Hankel). All functions of $D$ are Toeplitz plus Alternating Hankel. We explore this subspace of matrices. The slowly varying case (not quite Toeplitz) is also of interest.

### Determinantal representations of stable polynomials

Hugo Woerdeman

For every stable multivariable polynomial $p$, with $p(0) = 1$, we construct a determinantal representation

$$p(z) = \det(I_n - M(z)),$$

where $M(z)$ is a matrix vauled rational function with $\|M(z)\| \leqslant 1$ and $\|M(z)^n\| < 1$ for $z \in \mathbb{T}^d$ and $M(az) = aM(z)$ for all $a \in \mathbb{C} \setminus \{0\}$.

## Symbolic Matrix Algorithms

### Reducing memory consumption in Strassen-like matrix multiplication

Brice Boyer, Jean-Guillaume Dumas

Strassen–Winograd matrix multiplication algorithm is efficient in practice but needs temporary matrix allocations. We present techniques and tools used to reduce that need for extra memory in Strassen-like matrix-matrix multiplication, but also in the product with accumulation variant. The effort is put in keeping a time complexity close to Strassen–Winograd's (i.e. $6n^{\log_2(7)} - cn^2 + o(n^2)$) while using less extra space than what was proposed in the literature. We finally show experimental results and comparisons.

### Early Termination over Small Fields: A Consideration of the Block Case

Wayne Eberly

The incorporation of an "early termination" heuristic as part of a Wiedemann-style algorithm for a matrix computation can significantly reduce the cost to apply the algorithm on some inputs. The matrix properties that suffice to prove the reliability of such a heuristic — which also serve to bound the cost to use a Lanczos-style algorithm instead — hold generically. This makes it challenging to describe an input for which such a heuristic will *not* be reliable. Indeed, I have believed for several years that no such input exists. That is, I believe that these heuristics are reliable for all input matrices, over all fields, and using all blocking factors.

While it would still be necessary to condition inputs in order to achieve other matrix properties, related to the particular computation that is being performed, a proof of such a result would eliminate the need to condition (or choose blocking factors) in order to ensure the reliability of this kind of heuristic.

In this talk, I will discuss ongoing work to obtain such a proof.

### Outlier detection by error correcting decoding and structured linear algebra methods

Erich Kaltofen

Error-correcting decoding is generalized to multivariate sparse polynomial and rational function interpolation from evaluations that can be numerically inaccurate and where several evaluations can have severe errors ("outliers").

Our univariate solution for numerical Prony-Blahut / Giesbrecht-Labahn-Lee sparse interpolation can identify and remove outliers by a voting scheme.

Our multivariate polynomial and rational function interpolation algorithm combines Zippel's symbolic sparse polynomial interpolation technique [Ph.D. Thesis MIT 1979] with the numeric algorithm by Kaltofen, Yang, and Zhi [Proc. SNC 2007], and removes outliers ("cleans up data") by techniques from the Berlekamp/Welch decoder for Reed-Solomon codes.

Our algorithms can build a sparse function model from a number of evaluations that is linear in the sparsity of the model, assuming a constant number of ouliers.

This is joint work with Matthew Comer (NCSU), Clement Pernet (Univ. Grenoble), and Zhengfeng Yang (ECNU, Shanghai).

**Applications of fast nullspace computation for polynomial matrices**

George Labahn, Wei Zhou

In this talk we give a number of fast, deterministic algorithms for polynomial matrix problems. The algorithms include algorithms for computing matrix inverses, column bases, determinant and matrix normal forms such as Hermite and Popov. These algorithms all depend on fast algorithms for order bases (Zhou and Labahn in ISSAC 2009) and nullspace bases (Zhou, Labahn, Storjohann in ISSAC 2012). The computational costs are all similar to the cost of multiplying matrices of the same size as the input and having degrees equal to the average column degree of the input matrix.

**Polynomial Evaluation and Interpolation: Fast and Stable Approximate Solution**

Victor Pan

Multipoint polynomial evaluation and interpolation are fundamental for modern algebraic and numerical computing. The known algorithms solve both problems over any field by using $O(N \log^2 N)$ arithmetic operations for the input of size $N$, but the cost grows to quadratic for numerical solution. Our study results in numerically stable algorithms that use $O(uN \log N)$ arithmetic time for approximate evaluation (within the relative output error norm $2^{-u}$) and $O(uN \log^2 N)$ time for approximate interpolation. The problems are equivalent to multiplication of an $n \times n$ Vandermonde matrix by a vector and the solution of a nonsingular Vandermonde linear systems of $n$ equations, respectively. The algorithms and complexity estimates can be applied in both cases as well as where the transposed Vandermonde matrices replace Vandermonde matrices. Our advance is due to employing and extending our earlier method of the transformation of matrix structures, which enables application of the HSS–Multipole method to our tasks and further extension of the algorithms to more general classes of structured matrices.

**Wanted: reconditioned preconditioners**

David Saunders

Probabilistic preconditioners play a central role in blackbox methods for exact linear algebra. They typically introduce runtime and space cost factors that are logarithmic in the matrix dimension. This applies for example to blackbox algorithms for rank, solving systems, sampling nullspace, minimal polynomial.

However, the worst case scenarios necessitating the preconditioner overhead are rarely encountered in practice. This talk is a call for stronger and more varied analyses of preconditioners so that the practitioner has adequate information to engineer an effective implementation. I will offer a mixture of results, partial results, experimental evidence, and open questions illustrating several approaches to refining preconditioners and their analyses. I will discuss the block Wiedemann algorithm, diagonal preconditioners, and, if time permits, linear space preconditioners for rectangular matrices.

**Computing the invariant structure of integer matrices: fast algorithms into practice**

Arne Storjohann, Colton Pauderis

A lot of information about the invariant structure of a nonsingular integer matrix can be obtained by looking at random projections of the column space of the rational inverse. In this talk I describe an algorithm that uses such projections to compute the determinant of a nonsingular $n \times n$ matrix. The algorithm can be extended to compute the Hermite form of the input matrix. Both the determinant and Hermite form algorithm certify correctness of the computed results. Extensive empirical results from a highly optimized implementation show the running time grows approximately as $n^3 \log n$, even for input matrices with a highly nontrivial Smith invariant structure.

**Euclidean lattice basis reduction: algorithms and experiments for disclosing integer relations**

Gilles Villard

We review polynomial time approaches for computing simultaneous integer relations among real numbers. A variant of the LLL lattice reduction algorithm (A. Lenstra, H. Lenstra, L. Lovász, 1982), the HJLS (J. Hastad, B. Just, J. Lagarias, J. and C.P. Schnorr, 1989) and the PSLQ (H. Ferguson, D. Bailey, 1992) algorithms are de facto standards for solving the problem. We investigate the links between the various approaches and present intensive experiment results. We especially focus on the question of the precision for the underlying floating-point procedures used in the currently fastest known algorithms and software libraries. Part of this work is done in collaboration with D. Stehlé in relation with the fplll library, and with D. Stehlé and J. Chen for the HJLS/PSLQ comparison.

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## Contributed Session on Algebra and Matrices over Arbitrary Fields Part 1

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**Commutativity preserving maps via maximal cen-**

tralizers

Gregor Dolinar, Alexander Guterman, Bojan Kuzma, Polona Oblak

Mappings on different spaces which preserve commutativity and have some additional properties were studied by many authors. Usually one of these additional properties was linearity. Recently Šemrl characterized bijective maps on complex matrices that preserve commutativity in both directions without a linearity assumption.

We will present the characterization of bijective maps preserving commutativity in both directions on matrices over algebraically non-closed fields thus extending Šemrl's result. We obtained our result using recently published characterizations of matrices with maximal and with minimal centralizers over arbitrary fields with sufficiently many elements.

### Paratransitive algebras of linear transformations and a spatial implementation of "Wedderburn's Principal Theorem"

Leo Livshits, Gordon MacDonald, Laurent Marcoux, Heydar Radjavi

We study a natural weakening – which we refer to as *paratransitivity* – of the well-known notion of transitivity of an algebra $\mathcal{A}$ of linear transformations acting on a finite-dimensional vector space $\mathcal{V}$. Given positive integers $k$ and $m$, we shall say that such an algebra $\mathcal{A}$ is $(k,m)$-transitive if for every pair of subspaces $\mathcal{W}_1$ and $\mathcal{W}_2$ of $\mathcal{V}$ of dimensions $k$ and $m$ respectively, we have $\mathcal{A}\mathcal{W}_1 \cap \mathcal{W}_2 \neq \{0\}$. We consider the structure of minimal $(k,m)$-transitive algebras and explore the connection of this notion to a measure of largeness for invariant subspaces of $\mathcal{A}$.

To discern the structure of the paratransitive algebras we develop a spatial implementation of "Wedderburn's Principal Theorem", which may be of interest in its own right: if a subalgebra $\mathcal{A}$ of $\mathcal{L}(\mathcal{V})$ is represented by block-upper-triangular matrices with respect to a maximal chain of its invariant subspaces, then after an application of a block-upper-triangular similarity, the block-diagonal matrices in the resulting algebra comprise its Wedderburn factor. In other words, we show that, up to a block-upper-triangular similarity, $\mathcal{A}$ is a linear direct sum of an algebra of block-diagonal matrices and an algebra of strictly block-upper-triangular matrices.

### On extremal matrix centralizers

Gregor Dolinar, Alexander Guterman, Bojan Kuzma, Polona Oblak

Centralizers of matrices induce a natural preorder, where $A \preceq B$ if the centralizer of $A$ is included in the centralizer of $B$. Minimal and maximal matrices in this preorder are of special importance.

In the talk, we first recall known equivalent characterizations of maximal matrices and also of minimal matrices over algebraically closed fields. Then we present analogue characterizations for maximal matrices over arbitrary fields and minimal matrices over fields with sufficiently large cardinality.

### Adjacency preserving maps

Peter Semrl

Two $m \times n$ matrices $A$ and $B$ are adjacent if rank $(B - A) = 1$. Hua's fundamental theorem of geometry of rectangular matrices describes the general form of bijective maps on the set of all $m \times n$ matrices preserving adjacency in both directions. Some recent improvements of this result will be presented.

## Contributed Session on Algebra and Matrices over Arbitrary Fields Part 2

### On trivectors and hyperplanes of symplectic dual polar spaces

Bart De Bruyn, Mariusz Kwiatkowski

Let $V$ be a vector space of even dimension $2n \geqslant 4$ over a field $\mathbb{F}$ and let $f$ be a nondegenerate alternating bilinear form on $V$. The symplectic group $Sp(V,f) \cong Sp(2n,\mathbb{F})$ associated with the pair $(V,f)$ has a natural action on the $n$-th exterior power $\bigwedge^n V$ of $V$. Two elements $\chi_1$ and $\chi_2$ of $\bigwedge^n V$ are called $Sp(V,f)$*-equivalent* if there exists a $\theta \in Sp(V,f)$ such that $\chi_1^\theta = \chi_2$. The $Sp(V,f)$-equivalence classes which arise from this group action can subsequently be merged to obtain the so-called *quasi-equivalence classes*. With the pair $(V,f)$ there is also associated a point-line geometry $DW(2n-1,\mathbb{F})$ (called a *symplectic dual polar space*) and these quasi-equivalence classes form the basis of studying the so-called hyperplanes of that geometry. With a *hyperplane* we mean a set of points of the geometry meeting each line in either a singleton or the whole line.

The talk will focus on the case $n = 3$. In the special case that $n = 3$ and $\mathbb{F}$ is an algebraically closed field of characteristic distinct from 2, a complete classification of the $Sp(V,f)$-equivalence classes was obtained by Popov. In the talk, I will present a complete classification which is valid for any field $\mathbb{F}$. This (long) classification was realized in a number of papers, ultimately resulting in a description using 28 families (some of which are parametrized by elements of $\mathbb{F}$). If there is still time, I will also discuss the classification of the quasi-$Sp(V,f)$-equivalence classes and some applications to hyperplanes of $DW(2n-1,\mathbb{F})$.

### Uniform Kronecker Quotients

Yorick Hardy

Leopardi introduced the notion of a Kronecker quotient in 2005[1]. We consider the basic properties that a Kronecker quotient should satisfy and completely characterize a class of Kronecker quotients, namely uniform Kronecker quotients.

Not all properties of Kronecker products have corresponding properties for Kronecker quotients which are satisfiable. Results regarding some of these properties for uniform Kronecker quotients will be presented. Finally, some examples of uniform Kronecker quotients will be described.

[1] "A generalized FFT for Clifford algebras", P. Leopardi, Bulletin of the Belgian Mathematical Society, **11**, 663–688, 2005.

### Partial matrices of constant rank over small fields

Rachel Quinlan

In a *partial matrix* over a field $F$, entries are either specified elements of $F$ or indeterminates. Indeterminates in different positions are independent, and a *completion* of the partial matrix may be obtained by assigning a value from $F$ to each indeterminate. A number of recent research articles have investigated partial matrices whose completions all have the same rank or satisfy specified rank bounds.

We consider the following question: if all completions of a partial $m \times n$ matrix $A$ over a field $F$ have the same rank $r$, must $A$ possess a $r \times r$ sub(partial)matrix whose completions are all non-singular? This question can be shown to have an affirmative answer if the field $F$ has at least $r$ elements. This talk will concentrate on the case where this condition is not satisfied. We will show that the question above can have a negative answer if $|F| < r$ and $\max(m, n) \geqslant 2|F|$.

### Permanents, determinants, and generalized complementary basic matrices

Miroslav Fiedler, Frank Hall, Mikhail Stroev

In this talk we answer the questions posed in the article A note on permanents and generalized complementary basic matrices, Linear Algebra Appl. 436 (2012), by M. Fiedler and F. Hall. Further results on permanent compounds of generalized complementary basic matrices are obtained. Most of the results are also valid for the determinant and the usual compound matrix. Determinant and permanent compound products which are intrinsic are also considered, along with extensions to total unimodularity.

## Contributed Session on Computational Science

### Multigrid methods for high-dimensional problems with tensor structure

Matthias Bolten

Tensor structure is present in many applications, ranging from the discretization of possibly high dimensional partial differential equations to structured Markov chains. Many approaches have been studied recently to solve the associated systems, including Krylov subspace methods.

In this talk, we will present multigrid methods that can be used for certain structured problems. This is of interest when not only the dimension $d$ is large but also the basis $n$, where the number of degrees of freedoms is given by $N = n^d$. If the problem has $d$-level circulant or Toeplitz structure, the theoretical results for multigrid methods for these classes of matrices hold and further an efficient implementation for low-rank approximations is possible. Theoretical considerations as well as numerical results are presented.

### Discrete Serrin's Problem

Cristina Arauz, Angeles Carmona, Andres Encinas

In 1971, J. Serrin solved the following problem on Potential Theory: If $\Omega$ is a smooth bounded open connected domain in the Euclidean space for which the solution $u$ to the Dirichlet problem $\Delta u = -1$ in $\Omega$ and $u = 0$ on $\partial\Omega$ has overdetermined boundary condition $\dfrac{\partial u}{\partial n} =$ constant on $\partial\Omega$, then $\Omega$ is a ball and $u$ has radial symmetry. This result has multiple extensions, and what is more interesting, the methods used for solving the problem have applications to study the symmetry of the solutions of very general elliptic problems.

We consider here the discrete analogue problem. Specifically, given $\Gamma = (V, E)$ a graph and $F \subset V$ a connected subset, the solution of the Dirichlet problem $\mathcal{L}(u) = 1$ on $F$ and $u = 0$ on $\delta(F)$ is known as equilibrium measure. Therefore, the discrete Serrin's problem can be formulated as: does the additional condition $\dfrac{\partial u}{\partial n} =$ constant on $\delta(F)$ imply that $F$ is a ball and $u$ is radial? We provide some examples showing that the answer is negative. However, the values of the equilibrium measure depend on the distance to the boundary and hence we wonder if imposing some symmetries on the domain, the solution to Serrin's problem is radial. The conclusion is true for many families of graph with high degree of symmetry, for instance for distance–regular graphs or spider networks.

### A New Algebraic Approach to the Construction of Multidimensional Wavelet Filter Banks

Youngmi Hur, Fang Zheng

In this talk we present a new algebraic approach for constructing the wavelet filter bank. Our approach enables constructing nonseparable multidimensional non-redundant wavelet filter banks with a prescribed number of vanishing moments. Our construction method works under a very general setting–it works for any spatial dimension and for any sampling matrix–and, at the same time, it does not require the initial lowpass filters to satisfy any additional assumption such as interpolatory condition. In fact, our method is general enough to include some existing construction methods that work only for interpolatory lowpass filters, such as the multidimensional construction method developed using the traditional lifting scheme, as special cases.

### A new method for solving completely integrable PDEs

Andrey Melnikov

We will demonstrate a new method for solving (some) completely integrable PDEs. At the basic core-stone level this method include an analogue of the "scattering theory", which is implemented using a theory of vessels. Evolving such vessels we create solutions of completely integrable PDEs. For example, KdV and evolutionary NLS equation are the same evolutionary vessel for different vessel parameters. Using more sophisticated construction, we solve the Boussinesq equation.

This theory has connection to Zacharov-Shabath scheme, and effectively generalizes this scheme. We will see examples of classical solutions for different equations, implemented in the theory of vessels and some new results.

### Green matrices associated with Generalized Linear Polyominoes

Angeles Carmona, Andres Encinas, Margarida Mitjana

A Polyomino is an edge–conected union of cells in the planar square lattice. Polyominoes are very popular in mathematical recreations, and have found interest among mathematicians, physicists, biologists, and computer scientists as well. Because the chemical constitution of a molecule is conventionally represented by a molecular graph or network, the polyominoes have deserved the attention of the Organic Chemistry community. So, several molecular structure descriptors based in network structural descriptors, have been introduced. In particular, in the last decade a great amount of works devoted to calculate the Kirchhoff Index of linear polyominoes–like networks, have been published. In this work we deal with this class of polyominoes, that we call generalized linear polyominoes, that besides the most popular class of linear polyomino chains, also includes cycles, Phenylenes and Hexagonal chains to name only a few. Because the Kirchhoff Index is the trace of the Green function of the network, here we obtain the Green function of generalized linear Polyominoes. To do this, we understand a Polyomino as a perturbation of a path by adding weighted edges between opposite vertices. Therefore, unlike the techniques used by Yang and Zhang in 2008, that are based on the decomposition of the combinatorial Laplacian in structured blocks, here we obtain the Green function of a linear Polyomino from a perturbation of the combinatorial Laplacian. This approach deeply link linear Polyomino Green functions with the inverse $M$–matrix problem and specially, with the so–called Green matrices, of Gantmacher and Krein (1941).

### A Model Reduction Algorithm for Simulating Sedimentation Velocity Analysis

Hashim Saber

An algorithm for the construction of a reduced model is developed to efficiently simulate a partial differential equation with distributed parameters. The algorithm is applied to the Lamm equation, which describes the sedimentation velocity experiment. It is a large scale inverse model that is costly to evaluate repeatedly. Moreover, its high-dimensional parametric input space compounds the difficulty of effectively exploring the simulation process. The proposed parametric model reduction is applied to the simulating process of the sedimentation velocity experiment. The model is treated as a system with sedimentation and diffusion parameters to be preserved during model reduction. Model reduction allows us to reduce the simulation time significantly and, at the same time, it maintains a high accuracy.

## Contributed Session on Graph Theory

### A characterization of spectrum of a graph and of its vertex deleted subgraphs

Milica Andelic, Slobodan Simic, Carlos Fonseca

Let $G$ be a graph and $A$ its adjacency matrix. It is well known the relation between the multiplicity of $\lambda$ as an eigenvalue of $A$ and of the principal submatrix of $A$ obtained by deleting row and column indexed by a vertex of $G$. Here, we provide characterizations of the three possible types of vertices of $G$.

### Perturbations of Discrete Elliptic operators

Angeles Carmona, Andres Encinas, Margarida Mitjana

Given $V$ a finite set, a self–adjoint operator $\mathcal{K}$ is called elliptic if it is positive semi–definite and its lowest eigenvalue is simple. Therefore, there exists a unique, up to sign, unitary function $\omega \in \mathcal{C}(V)$ satisfying $\mathcal{K}(\omega) = \lambda\omega$ and then $\mathcal{K}$ is named $(\lambda, \omega)$–*elliptic*. Clearly, a $(\lambda, \omega)$–elliptic operator is singular iff $\lambda = 0$. Examples of elliptic operators are the so–called Schrödinger operators on finite connected networks, as well as the signless Laplacian of connected bipartite graphs.

If $\mathcal{K}$ is a $(\lambda, \omega)$–elliptic operator, it defines an automorphism on $\omega^{\perp}$, whose inverse is called *orthogonal Green operator of* $\mathcal{K}$. We aim here at studying the effect of a perturbation of $\mathcal{K}$ on its orthogonal Green operator. The perturbation here considered is performed by adding a self–adjoint and positive semi–definite operator to $\mathcal{K}$. As particular cases we consider the effect of changing the conductances on semi–definite Schödinger operators on finite connected networks and the signless Laplacian of connected bipartite graphs. The expression obtained for the perturbed networks is explicitly given in terms of the orthogonal Green function of the original network.

### Matrices and their Kirchhoff Graphs

Joseph Fehribach

Given a matrix with integer or rational elements, what graph or graphs represent this matrix in terms of the fundamental theorem of linear algebra? This talk will define the concept of a Kirchhoff or fundamental graph for the matrix and will explain how such a graph represents a matrix. A number of basic results pertaining to Kirchhoff graphs will be presented and a process for constructing them will be discussed. Finally the talk will conclude with an example from electrochemistry: the production of NaOH from NaCl. Kirchhoff graphs have their origin in electrochemical reaction networks and are closely related to reaction route graphs.

### Locating eigenvalues of graphs

David Jacobs, Vilmar Trevisan

We present a linear-time algorithm that determines the number of eigenvalues, in any real interval, of the adjacency matrix $A$ of a tree. The algorithm relies on Sylvester's Law of Inertia, and produces a diagonal matrix congruent to $A + xI$. Efficient approximation of a single eigenvalue can be achieved by divide-and-conquer.

The algorithm can be adapted to other matrices, including Laplacian matrices of trees. We report on several applications of the algorithm, such as integrality of trees, distribution and bounds of Laplacian eigenvalues, obtaining trees with extremal values of Laplacian energy, and ordering trees by Laplacian energy.

Finally, we present a diagonalization procedure for adjacency matrices of threshold graphs, also leading to a linear-time algorithm for computing the number of eigenvalues in a real interval. Applications include a formula for the smallest eigenvalue among threshold graphs of size n, a simplicity result, and a characteristic polynomial construction. A similar diagonalization algorithm is obtained for the distance matrix of threshold graphs.

### The Cycle Intersection Matrix and Applications to Planar Graphs

Caitlin Phifer, Woong Kook

Given a finite connected planar graph $G$ with oriented edges, we define the *Cycle Intersection matrix*, $C(G)$, as follows. Let $c_{ii}$ be the length of the cycle which bounds finite face $i$, and $c_{ij}$ be the (signed) number of edges that cycles $i$ and $j$ have in common. We give an elementary proof that the determinant of this matrix is equal to the number of spanning trees of $G$. As an application, we compute the number of spanning trees of the grid graph in terms of Chebychev polynomials. In addition, we show an interesting connection between the determinant of $C(G)$ to the Fibonacci sequence when $G$ is a triangulated $n-$gon. If time permits, we will also discuss the spanning tree entropy for grid graphs.

### Positive Semidefinite Propagation Time

Nathan Warnberg

Positive semidefinite (PSD) zero forcing on a simple undirected graph $G$ is based on the following color change rule: Let $B \subseteq V(G)$ be colored black and the rest of the vertices be colored white. Let $C_1, C_2, \ldots, C_k$ be the connected components of $G - B$. For any black vertex $b \in B \cup C_i$ that has exactly one white neighbor $w \in B \cup C_i$, change the color of $w$ to black. A minimum PSD zero forcing set is a set of black vertices of minimum cardinality that color the entire graph black. The PSD propagation time of a PSDZFS $B$ of graph $G$ is the minimum number of iterations of the color change rule needed to force all vertices of $G$ black, starting with the vertices in $B$ black. Minimum and maximum PSD propagation time are taken over all minimum PSD zero forcing sets. Extreme propagation times, $|G|-1$, $|G|-2$, will be discussed as well as graph family results.

## Contributed Session on Matrix Completion Problems

### Matrix Completion Problems

Gloria Cravo

In this talk we present several results in the area of the so-called Matrix Completion Problems, such as the description of the eigenvalues or the characteristic polynomial of a square matrix when some of its entries are prescribed. We give special emphasis to the situation where the matrix is partitioned into several blocks and some of these blocks are prescribed.

### Ray Nonsingularity of Cycle Chain Matrices

Yue Liu, Haiying Shan

Ray nonsingular (RNS) matrices are a generalization of sign nonsingular (SNS) matrices from the real field to complex field. The problem of how to recognize ray nonsingular matrices is still open. In this paper, we give an algorithm to recognize the ray nonsingularity for the so called "cycle chain matrices", whose associated digraphs are of certain simple structure. Furthermore, the cycle chain matrices that are not ray nonsingular are classified into two classes according to whether or not they can be majorized by some RNS cycle chain matrices, and the case when they can is characterized.

### Partial matrices whose completions all have the same rank

James McTigue

A partial matrix over a field $\mathbb{F}$ is a matrix whose entries are either elements of $\mathbb{F}$ or independent indeterminates. A completion of such a partial matrix is obtained by specifying values from $\mathbb{F}$ for the indeterminates. We determine the maximum possible number of indeterminates in an $m \times n$ partial matrix ($m \leqslant n$) whose completions all have a particular rank $r$, and we fully describe those examples in which this maximum is attained, without any restriction on the field $\mathbb{F}$.

We also describe sufficient conditions for an $m \times n$ partial matrix whose completions all have rank $r$ to contain an $r \times r$ partial matrix whose every completion is nonsingular.

This work extends some recent results of R. Brualdi, Z. Huang and X. Zhan.

### Dobrushin ergodicity coefficient for Markov operators on cones, and beyond

Stephane Gaubert, Zheng QU

The analysis of classical consensus algorithms relies on contraction properties of adjoints of Markov operators, with respect to Hilbert's projective metric or to a related family of seminorms (Hopf's oscillation or Hilbert's seminorm). We generalize these properties to abstract consensus operators over normal cones, which include the unital completely positive maps (Kraus operators) arising in quantum information theory. In particular, we show that the contraction rate of such operators, with respect to the Hopf oscillation seminorm, is given by an analogue of Dobrushin's ergodicity coefficient. We derive from this result a characterization of the contraction rate of a non-linear flow, with respect to Hopf's oscillation seminorm and to Hilbert's projective metric.

### The Normal Defect of Some Classes of Matrices

Ryan Wasson, Hugo Woerdeman

An $n \times n$ matrix $A$ has a normal defect of $k$ if there exists an $(n+k) \times (n+k)$ normal matrix $A_{ext}$ with $A$ as a leading principal submatrix and $k$ minimal. In this paper we compute the normal defect of a special class of $4 \times 4$ matrices, namely matrices whose only nonzero entries lie on the superdiagonal, and we provide details for constructing minimal normal completion matrices $A_{ext}$. We also prove a result for a related class of $n \times n$ matrices. Finally, we present an example of a $6 \times 6$ block diagonal matrix having the property that its normal defect is strictly less than the sum of the normal defects of each of its blocks, and we provide sufficient conditions for when the normal defect of a block diagonal matrix is equal to the sum of the normal defects of each of its blocks.

## Contributed Session on Matrix Equalities and Inequalities

### Rank One Perturbations of $H$-Positive Real Matrices

Jan Fourie, Gilbert Groenewald, Dawid Janse van Rensburg, Anre Ran

We consider a generic rank-one structured perturbation on $H$-positive real matrices. The complex case is treated in general, but the main focus for this article is the real case where something interesting occurs at eigenvalue zero and even size Jordan blocks. Generic Jordan structures of perturbed matrices are identified.

### Sylvester equation and some special applications

Hosoo Lee, Sejong Kim

In this article we mainly consider the following matrix equation for unknown $m \times n$ matrix $X$

$$A^{N-1}X + A^{N-2}XB + \cdots + AXB^{N-2} + XB^{N-1} = C, \tag{3}$$

where $A$ and $B$ are $m \times m$ and $n \times n$ matrices, respectively. First, we review not only the necessary and sufficient condition for the existence and uniqueness of the solution of Sylvester equation, but also the specific forms of the unique solution depending on the conditions of the spectra. Shortly we also study the special type of matrix equation associated with Lyapunov operator and see its unique solution derived from the known result of Sylvester equation. Finally we show that the equation (1) has a solution $X = AY - YB$ if $Y$ is a solution of the Sylvester equation $X = A^N X - XB^N$. Furthermore, we have the specific form of the unique solution of (1).

### Geometry of Hilbert's and Thompson's metrics on cones

Bas Lemmens, Mark Roelands

It is well-known that the cone of positive definite Hermitian matrices can be equipped with a Riemannian metric. There are two other important metrics on this cone, Hilbert's (projective) metric and Thompson's metric. These two metrics can be defined on general cones and their geometric properties are very interesting. For example, Hilbert's metric spaces are a natural non–Riemannian generalization of hyperbolic space. In this talk I will discuss some of the geometric properties of Hilbert's and Thompson's metrics. In particular, I will explain the structure of the unique geodesics and the possibility of quasi-isometric embeddings of these metric spaces into finite dimensional normed spaces.

### Refining some inequalities involving quasi–arithmetic means

Jadranka Micic Hot, Josip Pecaric, Kemal Hot

Let $\mathcal{A}$ and $\mathcal{B}$ be unital $C^*$-algebras on $H$ and $K$ respectively. Let $(x_t)_{t\in T}$ be a bounded continuous field of self-adjoint elements in $\mathcal{A}$ with spectra in an interval $[m, M]$, defined on a locally compact Hausdorff space $T$ equipped with a bounded Radon measure $\mu$. Let $(\phi_t)_{t\in T}$ be a unital field of positive linear mappings $\phi_t : \mathcal{A} \to \mathcal{B}$. We observe the quasi-arithmetic operator mean

$$\mathcal{M}_\varphi(\mathbf{x},\boldsymbol{\phi}) = \varphi^{-1}\left(\int_T \phi_t\left(\varphi(x_t)\right) d\mu(t)\right),$$

where $\varphi \in \mathcal{C}([m,M])$ is a strictly monotone function. This mean is briefly denoted by $\mathcal{M}_\varphi$. We denote $m_\varphi$ and $\mathcal{M}_\varphi$ the lower and upper bound of the mean $M_\varphi$, respectively, and $\varphi_m = \min\{\varphi(m), \varphi(M)\}$, $\varphi_M = \max\{\varphi(m), \varphi(M)\}$.

The purpose of this presentation is to consider refining converse inequalities involving quasi-arithmetic means. To obtain these results, we use refined converses of Jensen's inequality under the above assumptions. Applying the obtained results we further refine selected inequalities involving power means. In the end, a few numerical examples are given for various types of converse inequalities.

So, for example, we present the following ratio type converse order:

$$\begin{aligned}\mathcal{M}_\psi &\leqslant C(m_\varphi, M_\varphi, \varphi_m, \varphi_M, \psi\circ\varphi^{-1}, \psi^{-1}, \delta m_{\widetilde{x}})\,\mathcal{M}_\varphi\\ &\leqslant C(m_\varphi, M_\varphi, \varphi_m, \varphi_M, \psi\circ\varphi^{-1}, \psi^{-1}, 0)\,\mathcal{M}_\varphi\\ &\leqslant \overline{C}(\varphi_m, \varphi_M, \psi\circ\varphi^{-1}, \psi^{-1})\,\mathcal{M}_\varphi,\end{aligned}$$

for all strictly monotone functions $\varphi, \psi \in \mathcal{C}([m,M])$, such that $\varphi > 0$ on $[m,M]$, $\psi\circ\varphi^{-1}$ is convex and $\psi^{-1}$ is operator monotone, where we used the following abbreviations: $\delta \equiv \delta(m,M,\psi,\varphi) := \psi(m) + \psi(M) - 2\psi\circ\varphi^{-1}\left(\frac{\varphi(m)+\varphi(M)}{2}\right)$,

$m_{\widetilde{x}}$ is the lower bound of the operator

$\widetilde{x}(m,M,\varphi,\mathbf{x},\boldsymbol{\phi}) :=$

$\frac{1}{2}1_K - \frac{1}{|\varphi(M)-\varphi(m)|}\int_T \phi_t\left(\left|\varphi(x_t) - \frac{\varphi(m)+\varphi(M)}{2}1_H\right|\right) d\mu(t)$ and $C(n,N,m,M,f,g,c) := \max_{n\leqslant z\leqslant N}$

$$\left\{\frac{g\left(\frac{f(M)-f(m)}{M-m}z + \frac{Mf(m)-mf(M)}{M-m} - c\right)}{f\circ g(z)}\right\},$$

$\overline{C}(m,M,f,g) := C(m,M,m,M,f,g,0)$.

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## Contributed Session on Matrix Pencils

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### Symmetric Fiedler Pencils with Repetition as Strong Linearizations for Symmetric Matrix Polynomials.

<u>Maria Isabel Bueno Cachadina</u>, Kyle Curlett, Susana Furtado

Strong linearizations of an $n\times n$ matrix polynomial $P(\lambda) = \sum_{i=0}^k A_i\lambda^i$ that preserve some structure of $P(\lambda)$ are relevant in many applications. We characterize all the pencils in the family of the Fiedler pencils with repetition associated with a square matrix polynomial $P(\lambda)$ that are symmetric when $P(\lambda)$ is. The pencils in this family, introduced by Antoniou and Vologiannidis, are companion forms, that is, pencils whose matrix coefficients are block matrices whose blocks are of form $0_n$, $\pm I_n$, or $\pm A_i$. When some nonsingularity conditions are satisfied, these pencils are strong linearizations of $P(\lambda)$. In particular, when $P(\lambda)$ is a symmetric polynomial and the coefficients $A_k$ and $A_0$ are nonsingular, our family strictly contains the basis for the space $\mathbb{DL}(P)$ defined and studied by Mackey, Mackey, Mehl, and Mehrmann.

### Orbit closure hierarchies of skew-symmetric matrix pencils

<u>Andrii Dmytryshyn</u>, Stefan Johansson, Bo Kagstrom

The reduction of a complex skew-symmetric matrix pencil $A-\lambda B$ to any canonical form under congruence is an unstable operation: both the canonical form and the reduction transformation depend discontinuously on the entries of $A-\lambda B$. Thus it is important to know the canonical forms of all such pencils that are arbitrarily close to $A-\lambda B$, or in another words to know how small perturbations of a skew-symmetric matrix pencil may change its canonical form under congruence.

We solve this problem by deriving the *closure hierarchy graphs* (i.e., *stratifications*) of orbits and bundles of skew-symmetric matrix pencils. Each node of such a graph represents the orbit (or the bundle) and each edge represents the cover/closure relation (i.e., there is a directed path from the node $A-\lambda B$ to the node $C-\lambda D$ if and only if $A-\lambda B$ can be transformed by an arbitrarily small perturbation to a matrix pencil whose canonical form is $C-\lambda D$).

In particular, we prove that for two complex skew–symmetric matrix pencils $A-\lambda B$ and $C-\lambda D$ there exists a sequence of nonsingular matrices $\{R_n, S_n\}$ such that $R_n(C-\lambda D)S_n \to (A-\lambda B)$ if and only if there exists a sequence of nonsingular matrices $\{W_n\}$ such that $W_n^T(C-\lambda D)W_n \to (A-\lambda B)$. This result can be interpreted as a generalization (or a continuous counterpart) of the fact that two skew-symmetric matrix pencils are equivalent if and only if they are congruent.

### Skew-symmetric Strong Linearizations for Skew-symmetric Matrix Polynomials obtained from Fiedler Pencils with Repetition

<u>Susana Furtado</u>, Maria Isabel Bueno

Let $\mathbf{F}$ be a field with characteristic different from 2 and $P(\lambda)$ be a matrix polynomial of degree $k$ whose coefficients are matrices with entries in $\mathbf{F}$. The matrix polynomial $P(\lambda)$ is said to be skew-symmetric if $P(\lambda) = -P(\lambda)^T$.

In this talk we present $nk\times nk$ skew-symmetric matrix pencils associated with $n\times n$ skew-symmetric matrix polynomials $P(\lambda)$ of degree $k\geqslant 2$ whose coefficient matrices have entries in $\mathbf{F}$. These pencils are of the form $SL$, where $L$ is a Fiedler pencil with repetition and $S$ is a direct sum of blocks of the form $I_n$ or $-I_n$. Under certain conditions, these pencils are strong linearizations of $P(\lambda)$. These linearizations are companion forms in the sense that, if their coefficients are viewed as $k$-by-$k$ block matrices, each $n\times n$ block is either 0, $\pm I_n$, or $\pm A_i$, where $A_i$, $i = 0, \dots, k$, are the coefficients of $P(\lambda)$. The family of Fiedler pencils with repetition was introduced by S. Vologiannidis and E. N. Antoniou (2011).

### New bounds for roots of polynomials from Fiedler companion matrices

Froilan M. Dopico, Fernando De Teran, Javier Perez

In 2003 Miroslav Fiedler introduced a new family of companion matrices of a given monic scalar polynomial $p(z)$. This family of matrices includes as particular cases the classical Frobenius companion forms of $p(z)$. Every Fiedler matrix shares with the Frobenius companion forms that its characteristic polynomial is $p(z)$, that is, the roots of $p(z)$ are the eigenvalues of the Fiedler matrices. Frobenius companion matrices have been used extensively to obtain upper and lower bounds for the roots of a polynomial. In this talk, we derive new upper and lower bounds of the absolute values of the roots of a monic polynomial using Fiedler matrices. These bounds are based in the fact that if $\lambda$ is an eigenvalue of a nonsingunlar matrix $A$ and $\|\cdot\|$ is a submultiplicative matrix norm, then $||A^{-1}||^{-1} \leqslant |\lambda| \leqslant ||A||$. For this purpose, we present explicit formulae for the 1-norm and $\infty$-norm of any Fiedler matrix and its inverse. These formulas will be key to obtain new upper and lower bounds for the roots of monic polynomials. In addition, we compare these new bounds with the bounds obtained from the Frobenius companion forms and we show that, for many polynomials, the lower bounds obtained from the inverses of Fiedler matrices improve the lower bounds obtained from Frobenius companion forms when $|p(0)| < 1$.

### Linearizations of Matrix Polynomials in Bernstein Basis

D. Steven Mackey, Vasilije Perovic

Scalar polynomials in the Bernstein basis are well studied due to their importance in applications, most notably in computer-aided geometric design. But, matrix polynomials expressed in the Bernstein basis have only recently been studied. Two considerations led us to systematically study such matrix polynomials: the increasing use of non-monomial bases in practice, and the desirable numerical properties of the Bernstein basis. For a matrix polynomial $P(\lambda)$, the classical approach to solving the polynomial eigenvalue problem $P(\lambda)x = 0$ is to first convert $P$ into a matrix pencil $L$ with the same finite and infinite elementary divisors, and then work with $L$. This method has been extensively developed for polynomials $P$ expressed in the monomial basis. But what about when $P(\lambda)$ is in the Bernstein basis? It is important to avoid reformulating such a $P$ into the monomial basis, since this change of basis is poorly conditioned, and can therefore introduce numerical errors that were not present in the original problem. Using novel tools, we show how to work directly with any $P(\lambda)$ expressed in the Bernstein basis to generate *large* families of linearizations that are also expressed in the Bernstein basis. Connections with low bandwidth Fiedler-like linearizations, which could have numerical impact, are also established. We also illustrate how existing structure preserving eigenvalue algorithms for structured pencils in the monomial basis can be adapted for use with structured pencils in the Bernstein basis. Finally, we note that several results in the literature are readily obtained by specializing our techniques to the case of scalar polynomials expressed in the Bernstein basis.

### Perturbations of singular hermitian pencils.

Michal Wojtylak, Christian Mehl, Volker Mehrmann

Low rank perturbations of the pencils $A + \lambda E$, with hermitian-symmetric $A$ and $E$, are studied. The main interest lies in the case when the pencil is singular, also a distance of an arbitrary pencil to a singular pencil is studied.

## Contributed Session on Nonnegative Matrices Part 1

### About negative zeros of entire functions, totally non-negative matrices and positive definite quadratic forms

Prashant Batra

Holtz and Tyaglov in SIAM Review 54(2012) proposed recently a number of results characterising reality / negativity / interlacing of polynomial roots via totally non-negative, infinite matrices. We show how to replace such characterisations by structurally similar, finite equivalents, and subsequently extend the new criteria naturally from polynomials to entire functions with only real zeros. The extension relies on a three-fold connection between mapping properties of meromorphic functions, certain Mittag-Leffler expansions and non-negative Hankel determinants. We derive from this connection a simple, concise proof of an extended Liénard-Chipart criterion for polynomials and entire functions of genus zero. Using some infinite matrix $H$ for a real entire function $f$ of genus zero, we derive a new criterion for zero-location exclusively on the negative real axis. We show moreover that in the polynomial case (where $\deg f = n$) the first $n$ consecutive principal minors of even order already characterize total non-negativity of $H$.

### The nonnegative inverse eigenvalue problem

Anthony Cronin

In this talk we investigate the nonnegative inverse eigenvalue problem (NIEP). This is the problem of characterizing all possible spectra of nonnegative $n \times n$ matrices. In particular, given a list of $n$ complex numbers $\sigma = (\lambda_1, \lambda_2, ..., \lambda_n)$, can we find necessary and sufficient conditions on the list $\sigma$ so that it is the list of eigenvalues of an entry-wise $n \times n$ nonnegative matrix $A$. The problem remains unsolved and a complete solution is known only for n 4. I will give a brief outline of the history and motivation behind the NIEP and discuss the more important results to date. I will then give a survey of the main results from my PhD thesis and current work.

### An extension of the dqds algorithm for totally nonnegative matrices and the Newton shift

Akiko Fukuda, Yusaku Yamamoto, Masashi Iwasaki, Emiko Ishiwata, Yoshimasa Nakamura

The recursion formula of the qd algorithm is known as being just equal to the integrable discrete Toda equation. In this talk, we consider the discrete hungry Toda (dhToda) equation, which is a generalization of the discrete Toda equation. We show that the dhToda equation is applicable for computing eigenvalues of a class of totally nonnegative (TN) matrices where all of its minors are nonnegative. The algorithm is named the dhToda algorithm, and can be regarded as an extension of the dqds algorithm. The differential form and the origin shift are incorporated into the dhToda algorithm. It is also shown that the order of convergence for the dhToda algorithm with Newton shift is quadratic.

### An Iterative Method for detecting Semi-positive Matrices

Keith Hooper, Michael Tsatsomeros

Definition: A Semi-positive matrix, A, is a matrix such that there exists $x > 0$ satisfying $Ax > 0$. This class of matrices generalizes the well studied P matrices. It is known that determining whether or not a given matrix is a P matrix is co-NP complete. One of the questions we answer is whether or not this difficulty also arises in determining whether or not a matrix is Semi-positive. The simple definition of a Semi-positive matrix also has geometric interpretations, which will be mentioned briefly. In the talk, an iterative for detecting whether or not a matrix is Semi-positive is discussed.

### On Inverse-Positivity of Sub-direct Sums of Matrices

Shani Jose, Sivakumar K. C.

Sub-direct sum, a generalization of the direct sum and the normal sum of matrices, was proposed by Fallat and Johnson. This concept has applications in matrix completion problems and overlapping subdomains in domain decomposition methods. The definition of $k$-subdirect sum is given as follows:

Let $A = \begin{pmatrix} D & E \\ F & G \end{pmatrix}$ and $B = \begin{pmatrix} P & Q \\ S & T \end{pmatrix}$, where $D \in \mathbb{R}^{(m-k)\times(m-k)}$, $E \in \mathbb{R}^{(m-k)\times k}$, $F \in \mathbb{R}^{k\times(m-k)}$, $Q \in \mathbb{R}^{k\times(n-k)}$, $S \in \mathbb{R}^{(n-k)\times k}$, $T \in \mathbb{R}^{(n-k)\times(n-k)}$ and $G, P \in \mathbb{R}^{k\times k}$. The *k-subdirect* sum of $A$ and $B$ is denoted as $A \oplus_k B$ and is defined as

$$A \oplus_k B = \begin{pmatrix} D & E & 0 \\ F & G+P & Q \\ 0 & S & T \end{pmatrix}. \tag{4}$$

One of the main questions in connection with the sub-direct sum is to investigate whether certain special classes of matrices are closed with respect to the sub-direct sum operation. In this talk, we consider the class of all inverse-positive matrices. We provide certain conditions under which the $k$-subdirect sum of inverse-positive matrices is inverse-positive. We also present sufficient conditions for the converse to hold. In other words, if a matrix is inverse-positive, we prove that it can be written as a $k$-subdirect sum of inverse-positive matrices, in the presence of certain assumptions. We consider the case $k \neq 1$ as for the 1-subdirect sum, these results are known. We tackle the problem in its full generality by assuming that the individual summand matrices have non-trivial blocks.

### Matrix roots of positive and eventually positive matrices.

Pietro Paparella, Judith McDonald, Michael Tsatsomeros

Eventually positive matrices are real matrices whose powers become and remain strictly positive. As such, eventually positive matrices are *a fortiori* matrix roots of positive matrices, which motivates us to study the matrix roots of primitive matrices. Using classical matrix function theory and Perron-Frobenius theory, we characterize, classify, and describe in terms of the real Jordan canonical form the $p$th-roots of eventually positive matrices.

## Contributed Session on Nonnegative Matrices Part 2

### Product distance matrix of a tree with matrix weights

Ravindra Bapat, Sivaramakrishnan Sivasubramanian

Let $T$ be a tree with $n$ vertices and let $k$ be a positive integer. Let each edge of the tree be assigned a weight, which is a $k \times k$ matrix. The product distance from vertex $i$ to vertex $j$ is defined to be the identity matrix of order $k$ if $i = j$, and the product of the matrices corresponding to the $ij$-path, otherwise. The distance matrix of $T$ is the $nk \times nk$ block matrix with its $ij$-block equal to the product distance from $i$ to $j$. We obtain formulas for the determinant and the inverse of the product distance matrix, generalizing known results for the case $k = 1$.

### Some monotonicity results for Euclidean distance matrices

Hiroshi Kurata

In this talk, some monotonicity results for Euclidean distance matrices (EDMs) are given. We define a partial ordering on the set of EDMs and derive several inequalities for the eigenvalues and the set of configuration of the EDMs. The talk can be viewed as some further results of Kurata and Sakuma (2007, LAA) and Kurata and Tarazaga (2012, LAA).

**On the CP-rank and the DJL Conjecture**

Naomi Shaked-Monderer, Imanuel Bomze, Florian Jarre, Werner Schachinger, Abraham Berman

A matrix $A$ is *completely positive* if it can be factored as $A = BB^T$, where $B$ is a nonnegative, not necessarily square, matrix. The minimal number of columns in such $B$ is the *cp-rank* of $A$. Finding a sharp upper bound on the cp-rank of $n \times n$ completely positive matrices is one of the basic open questions in the study of completely positive matrices. According to the DJL conjecture (Drew, Johnson and Loewy, 1994), for $n \geqslant 4$ this bound may be $\lfloor n^2/4 \rfloor$. The best known bound for $n \geqslant 5$ has been until now the BB bound $n(n+1)/2 - 1$ (due to Barioli and Berman, 2003).

We show that the maximum cp-rank of $n \times n$ completely positive matrices is attained at a positive definite matrix on the boundary of the cone of $n \times n$ completely positive matrices, thus answering a long standing question. This new result is used to prove the DJL conjecture in the case $n = 5$, completing the work of Loewy and Tam (2003) on that case. For $n > 5$ we show that the maximum cp-rank of $n \times n$ completely positive is strictly smaller than the BB bound.

Based on joint works with Imanuel M. Bomze, Florian Jarre, Werner Schachinger and Abraham Berman.

**Binary factorizations of the matrix of all ones**

Maguy Trefois, Paul Van Dooren, Jean-Charles Delvenne

We consider the factorization problem of the square matrix of all ones where the factors are square matrices and constrained to have all elements equal to 0 or 1. We show that under some conditions on the rank of the factors, these are essentially De Bruijn matrices. More specifically, we are looking for the $n \times n$ binary solutions of $\prod_{i=1}^m A_i = \mathbb{I}_n$, where $\mathbb{I}_n$ is the $n \times n$ matrix with all ones and in particular, we are investigating the binary solutions to the equation $A^m = \mathbb{I}_n$. Our main result states that if we impose all factors $A_i$ to be $p_i$-regular and such that all their products commute, then $\text{rank}(A_i) = n/p_i$ if and only if $A_i$ is essentially a De Bruijn matrix. In particular, we prove that if $A$ is a binary $m$-th root of $\mathbb{I}_n$, then $A$ is $p$-regular. Moreover, $\text{rank}(A) = n/p$ if and only if $A$ is essentially a De Bruijn matrix.

**Nonnegative Eigenvalues and Total Orders**

Rohan Hemasinha, James Weaver

A solution to Problem 49-4 from "IMAGE" Issue Number 49, Fall 2012 and proposed by Rajesh Pereira is outlined. This problem is as follows: Let $A$ be an $n \times n$ real matrix. Show that the following are equivalent: (a) All the eigenvalues of $A$ are real and nonnegative. (b) There exists a total order @ on $R^n$ (partial order where any two vectors are comparable) which is preserved by $A$ (i.e. $x$ belongs to $R^n$ and $x$ @ 0 implies $Ax$ @ 0) and which makes ($R^n$,@) an ordered vector space. We then explore whether the total order is unique up to an isomorphism.

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**Contributed Session on Numerical Linear Algebra Part 1**

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**Block Gram-Schmidt Downdating**

Jesse Barlow

The problem of deleting multiple rows from a Q-R factorization, called the block downdating problem, is considered. The problem is important in the context of solving recursive least squares problem where observations are added or deleted over time.

The problem is solved by a block classical Gram-Schmidt procedure with re-orthogonalization. However, two heuristics are needed for the block downdating problem that are not needed for the cases of deleting a single row; one is a condition estimation heuristic, the other is a method for constructing a one left orthogonal matrix that is orthogonal to another. The difficulties that require these heuristic tend to occur when the block downdating operation changes the rank of the matrix.

**Structured eigenvalue condition numbers for parameterized quasiseparable matrices**

Froilán Dopico

It is well-known that $n \times n$ quasiseparable matrices can be represented in terms of $O(n)$ parameters or generators, but that these generators are not unique. In this talk, we present eigenvalue condition numbers of quasiseparable matrices with respect to tiny relative perturbations of the generators and compare these condition numbers for different specific set of generators of the same matrix.

**Explicit high-order time stepping based on componentwise application of asymptotic block Lanczos iteration**

James Lambers

This talk describes the development of explicit time stepping methods for linear and nonlinear PDEs that are specifically designed to cope with the stiffness of the system of ODEs that results from spatial discretization. As stiffness is caused by the contrasting behavior of coupled components of the solution, it is proposed to adopt a componentwise approach in which each coefficient of the solution in an appropriate basis is computed using an individualized approximation of the solution operator. This is accomplished by Krylov subspace spectral (KSS) methods, which use techniques from "matrices, moments and quadrature" to approximate bilinear forms involving functions of matrices via block Gaussian quadrature rules. These forms correspond to coefficients with respect to the chosen basis of the application of the solution operator of the PDE to the solution at an earlier time. In this talk, it is shown that the efficiency of KSS methods can be substantially enhanced through the prescription of quadrature nodes on the basis of asymptotic analysis of the recursion coefficients produced by block Lanczos iteration, for each Fourier coefficient as a function of frequency. The effectiveness of this idea is illustrated through numerical results obtained from the application of the modified KSS methods to diffusion equations and wave equations, as well as nonlinear equations through combination with exponential propagation iterative (EPI) methods.

### Gauss-Jordan elimination method for computing outer inverses

Marko Petkovic, Predrag Stanimirovic

This paper deals with the algorithm for computing outer inverse with prescribed range and null space, based on the choice of an appropriate matrix $G$ and Gauss-Jordan elimination of the augmented matrix $[G \mid I]$. The advantage of such algorithms is the fact that one can compute various generalized inverses using the same procedure, for different input matrices. In particular, we derive representations of the Moore-Penrose inverse, the Drazin inverse as well as $\{2,4\}$ and $\{2,3\}$-inverses. Numerical examples on different test matrices are presented, as well as the comparison with well-known methods for generalized inverses computation.

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## Contributed Session on Numerical Linear Algebra Part 2

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### Generalized Chebyshev polynomials

Clemente Cesarano

we present some generalizations of first and second kind Chebyshev polynomials by using the operational relations stated for Hermite polynomials. we derive integral representatons and new identity for two-variable Chebyshev polynomials. finally, we also show some relations for generalized Gegenbauer polynomials.

### CP decomposition of Partial symmetric tensors

Na Li, Carmeliza Navasca

We study the CANDECOMP/PARAFAC decomposition on partial symmetric tensor and present a new alternating way to solve the factor matrices. The original objective function is reformulated and the factor matrices involving in the symmetric modes are updated column by column. We also provide the numerical examples to show that our method is better than the classical alternating least squares method in terms of the CPU time and number of iterations.

### On Moore-Penrose Inverse based Image Restoration

Surya Prasath, K. C. Sivakumar, Shani Jose, K. Palaniappan

We consider the motion deblurring problem from image processing. A linear motion model along with random noise is assumed for the image formation process. We solve the inverse problem of estimating the latent image using the Moore-Penrose (MP) inverse of the blur matrix. Traditional MP inverse based schemes can lead to ringing artifacts due to approximate computations involved. Here we utilize the classical Grevile's partitioning based method [T. N. E. Grevile, Some applications of the pseudoinverse of matrix, SIAM Review 3(1), 1522, 1960.] for computing the MP inverse of the deblurring operator along with a Tikhonov regularization for the denoising step. We provide a review of MP inverse based image processing techniques and show some experimental results in image restoration.

### Construction of all discrete orthogonal and $q$-orthogonal classical polynomial sequences using infinite matrices and Pincherle derivatives

Luis Verde-Star

In the paper: L. Verde-Star, Characterization and construction of classical orthogonal polynomials using a matrix approach, Linear Algebra Appl. (2013), `http://dx.doi.org/10.1016/j.laa.2013.01.014`, we obtained explicit formulas for the recurrence coefficients and the Bochner differential operator of all the classical orthogonal polynomial sequences. In this talk we present some extensions and simplifications of our previous results that allow us to find, in an explicit way, all the discrete orthogonal and $q$-orthogonal classical polynomial sequences.

We use Pincherle derivatives to solve a system of two infinite matrix equations of the form $LA = AX$, $YAE = AB$, where $A$ is the matrix associated with an orthogonal sequence of polynomials $p_k(x)$, $L$ is the tridiagonal matrix of recurrence coefficients, $X$ is the representation of multiplication by the variable $x$, $Y$ is a diagonal matrix, $B$ represents a Bochner-type operator, and $E$ is the advance operator.

The Pincherle derivative of a matrix $Z$ is defined by $Z' = XZ - ZX$. Using Pincherle derivatives we show that $BE^{-1}$ and $X$ satisfy a simple polynomial equation that must be satisfied also when $BE^{-1}$ is replaced by $Y$ and $X$ is replaced by $L$. All the entries of the matrices are found in terms of 4 of the initial recurrence coefficients and a scaling parameter that acts on $Y$ and $B$.

### An Algorithm for the Nonlinear Eigenvalue Problem based on the Contour Integral

Yusaku Yamamoto

We consider the solution of the nonliear eigenvalue problem $A(z)\mathbf{x} = \mathbf{0}$, where $A(z)$ is an $n \times n$ matrix whose elements are analytical functions of a complex parameter $z$, and $\mathbf{x}$ is an $n$-dimensional vector. In this talk, we focus on finding all the eigenvalues that lie in a specified region on the complex plane.

Conventional approaches for the nonlinear eigenvalue problem include multivariate Newton's method and its modifications, nonlinear Arnoldi methods and nonlinear Jacobi-Davidson methods. However, Newton's method requires a good initial estimate for stable convergence. Nonlinear Arnoldi and Jacobi-Davidson methods are efficient for large sparse matrices, but in general, they cannot guarantee that all the eigenvalues in a specified region are obtained.

In our approach, we construct a complex function $g(z)$ that has simple poles at the roots of the nonlinear eigenvalue problem and is analytical elsewhere. This can be achieved by letting $f(z) = \det(A(z))$ and setting $g(z)$ as

$$g(z) \equiv \frac{f'(z)}{f(z)} = \mathrm{Tr}\left[(A(z))^{-1}\,\frac{dA(z)}{dz}\right]. \qquad (5)$$

Then, by computing the complex moments through contour integration along the boundary of the specified region, we can locate *all* the poles, or eigenvalues, using the method of Kravanja et al. The multiplicity of each eigenvalue can also be obtained from the 0th moment. Another advantage of our approach is its coarse-grain parallelism. This is because the main computational task is evaluation of $g(z)$ at sample points along the integration path and each evaluation can be done independently.

A possible difficulty in this approach is that the evaluation of $g(z)$ can be costly, because it involves matrix inverse $(A(z))^{-1}$ and its product with another matrix. To solve this problem, we developed an efficient algorithm to compute $g(z)$ by extending Erisman and Tinney's algorithm to compute $\mathrm{Tr}\left[A^{-1}\right]$. Given $\frac{dA(z)}{dz}$ and the $LU$ factors of $A(z)$, our algorithm can compute $g(z)$ with only twice the work needed to compute the $LU$ decomposition of $A(z)$. This is very efficient when $A(z)$ is a large sparse matrix.

We implemented our algorithm and evaluated its performance on the Fujitsu HPC2500 supercomputer. For random matrices of size from 500 to 2,000 with exponential dependence on $z$, our algorithm was able to find all the eigenvalues within the unit circle centered at the origin with high accuracy. The parallel speedup was nearly linear, as can be expected from the structure of the algorithm.

## Contributed Session on Numerical Linear Algebra Part 3

### A Geometrical Approach to Finding Multivariate Approximate LCMs and GCDs

Kim Batselier, Philippe Dreesen, Bart De Moor

In this talk we present a new approach to numerically compute an approximate least common multiple (LCM) and an approximate greatest common divisor (GCD) of two multivariate polynomials. This approach uses the geometrical notion of principal angles between subspaces whereas the main computational tools are Implicitly Restarted Arnoldi Iterations, the SVD and the QR decomposition. Upper and lower bounds are derived for the largest and smallest singular values of the highly structured Macaulay matrix. This leads to an upper bound on its condition number and an upper bound on the 2-norm of the product of two multivariate polynomials. Numerical examples illustrate the effectiveness of the proposed approach.

### Global Krylov subspace methods for computing the meshless elastic polyharmonic splines

Abderrahman Bouhamidi

Meshless elastic polyharmonic splines are useful for the approximation of vector fields from scattered data points without using any mesh nor grid. They are based on a minimization of certain energy in an appropriate functional native space. A such energy is related to the strain tensor constraint and to the divergence of the vector field. The computation of such splines leads to a large linear system. In this talk, we will discuss how to transform a such linear system to a general Sylvester matrix equation. So, we will use global Krylov subspace methods to compute approximations to the solution.

### The Stationary Iterations Revisited

Xuzhou Chen, Xinghua Shi, Yimin Wei

In this talk, we first present a necessary and sufficient conditions for the weakly and strongly convergence of the general stationary iterations $x^{(k+1)} = Tx^{(k)} + c$ with initial iteration matrix $T$ and vectors $c$ and $x^{(0)}$. Then we apply these general conditions and present convergence conditions for the stationary iterations for solving singular linear system $Ax = b$. We show that our convergence conditions are weaker and more general than the known results.

### Derivatives of Functions of Matrices

Sonia Carvalho, Pedro Freitas

There have been recent results concerning directional derivatives, of degree higher than one, for the determinant, the permanent, the symmetric and the antisymmetric power maps, by R. Bhatia, P. Grover and T. Jain. We present generalizations of these results for immanants and other symmetric powers of a matrix.

### Matrix-Free Methods for Verifying Constrained Positive Definiteness and Computing Directions of Negative Curvature

William Morrow

In this talk we review several methods for verifying *constrained* positive definiteness of a $N \times N$ matrix $\mathbf{H}$: positive definiteness over a linear subspace $\mathcal{C}$ of dimension $L$. $\mathcal{C}$ is often defined through the null space of some $M \times N$ matrix $\mathbf{A}$ ($M < N$, $N = M + L$). A related problem concerns computing a direction of negative curvature $\mathbf{d}$ for $\mathbf{H}$ in $\mathcal{C}$, defined as any $\mathbf{d} \in \mathcal{C}$ such that $\mathbf{d}^\top \mathbf{H}\mathbf{d} < 0$. This problem is primarily relevant in optimization theory, but also closely related to more general saddle-point systems; the computation of a direction of negative curvature, should one exist, is particularly important for second-order convergent optimization methods for nonconvex problems.

We present several contributions. First, existing methods based on Gaussian Elimination or inertia-revealing factorization require knowing $\mathbf{H}$ explicitly and do not reveal a direction of negative curvature. We present three new methods based on Cholesky factorization, orthogonalization, and (projected) Conjugate Gradients that are "matrix-free" in the sense that in place of $\mathbf{H}$ they require a routine to compute $\mathbf{Hv}$ for any $\mathbf{v}$ *and* reveal directions of negative curvature, should constrained positive definiteness not hold. Partitioned implementations that take advantage of the Level-3 `BLAS` routines are given and compared to existing methods using state of the art software (including `LAPACK`, `LUSOL`, `HSL`, and `SuiteSparse` routines). Second, we provide a variety of examples based on matrices from the UF sparse matrix collection as well as large-scale optimization test problems in which the appropriate $\mathbf{H}$ matrix is not known exactly. These results show that (a) different methods are better suited for different problems, (b) the ability to check the constrained positive definiteness of $\mathbf{H}$ (and compute a direction of negative curvature) when only products $\mathbf{Hv}$ are known is valuable to optimization methods, and (c) that numerical errors can lead to incorrect rejection of constrained positive definiteness. Third, we examine numerical accuracy of different methods, including cases where $\mathbf{H}$ itself or products $\mathbf{Hv}$ may contain errors. Practical problems must be founded on an understanding of how errors in computing null space bases and $\mathbf{H}$ (or products $\mathbf{Hv}$) obscure inferences of the constrained positive definiteness of the "true" $\mathbf{H}$. Our analysis is focused on practical methods that can certify inferences regarding the constrained positive definiteness of the "true" $\mathbf{H}$.

### A practical rational Krylov algorithm for solving large-scale nonlinear eigenvalue problems

Roel Van Beeumen, Karl Meerbergen, Wim Michiels

In a previous contribution, we introduced the Newton rational Krylov method for solving the nonlinear eigenvalue problem (NLEP):

$$A(\lambda)x = 0.$$

Now, we present a practical two-level rational Krylov algorithm for solving large-scale NLEPs. At the first level, the large-scale NLEP is projected yielding a small NLEP at the second level. Therefore, the method consists of two nested iterations. In the outer iteration, we construct an orthogonal basis $V$ and project $A(\lambda)$ onto it. In the inner iteration, we solve the small NLEP $\widehat{A}(\lambda)x = 0$ with $\widehat{A}(\lambda) = V^* A(\lambda) V$. We use polynomial interpolation of $\widehat{A}(\lambda)$ resulting, after linearization, in a generalized linear eigenvalue problem with a companion-type structure. The interpolating matrix polynomial is connected with the invariant pairs in such a way it allows locking of invariant pairs and implicit restarting of the algorithm. We also illustrate the method with numerical examples and give a number of scenarios where the method performs very well.

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## Contributed Session on Numerical Range and Spectral Problems

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### An exact method for computing the minimal polynomial

Christopher Bailey

We develop a method to obtain the minimal polynomial for any matrix. Our methods uses exact arithmetic and simplifies greatly when applied to lower Hessenberg matrices. Although numerical considerations are not considered, the algorithm may be easily implemented to be numerically efficient.

### Inclusion theorems for pseudospectra of block triangular matrices

Michael Karow

The $\epsilon$-pseudospectrum of $A \in \mathbb{C}^{n\times n}$, denoted by $\sigma_\epsilon(A)$, is the union of the spectra of the matrices $A + E$, where $E \in \mathbb{C}^{n\times n}$ and $\|E\|_2 \leqslant \epsilon$. In this talk we consider inclusion relations of the form $\sigma_{f(\epsilon)}(A_{11}) \cup \sigma_{f(\epsilon)}(A_{22}) \subseteq \sigma_\epsilon\left(\begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22}\end{bmatrix}\right) \subseteq \sigma_{g(\epsilon)}(A_{11}) \cup \sigma_{g(\epsilon)}(A_{22})$. We derive formulae for $f(\epsilon)$ and $g(\epsilon)$ in terms of $\mathrm{sep}_\lambda(A_{11}, A_{22})$ and $\|R\|_2$, where $R$ is the solution of the Sylvester equation $A_{11}R - RA_{22} = A_{12}$.

### Connectedness, Hessians and Generalized Numerical Ranges

Xuhua Liu, Tin-Yau Tam

The classical numerical range of a complex square matrix $A$ is the set

$$W(A) = \{x^*Ax : x \in \mathbb{C}^n \text{ and } \|x\|_2 = 1\},$$

which is convex. We give a brief survey on the convexity of some generalized numerical ranges associated with semisimple Lie algebras. We provide another proof of the convexity of a generalized numerical range associated with a compact Lie group via a connectedness result of Atiyah and a Hessian index result of Duistermaat, Kolk and Varadarajan.

### Matrix canonical forms under unitary similarity transformations. Geometric approach.

Yuri Nesterenko

Matrices $A, B \in C^{n\times n}$ are unitarily similar if a similarity transformation between them can be implemented using a unitary matrix $U$: $B = UAU^*$. Since unitary transformations preserve some important characteristics of a matrix (e.g. condition number) this class of matrix transformations is very popular in applications. We present method of verification of nonderogatory matrices for unitary similarity based on geometric approach. Given an arbitrary nonderogatory matrix, we construct a finite family of unitarily similar matrices for it. Whether or not two matrices are unitarily similar can be answered by verifying the intersection of their corresponding families. From the geometric point of view this canonical family is the finite set of the ruled surfaces in the space of upper triangular matrices.

As a rule, based on the Schur upper triangular form, different methods "tries" to create as many positive real off-diagonal elements as possible. But such a property of a desired canonical form inevitably leads to the form unstable with respect to errors in initial triangular form. Our approach is free of this shortcoming. The stability property seems to be even more important due to the fact that usually an upper triangular form of a matrix is obtained by approximate methods (e.g. QR algorithm).

## Contributed Session on Positive Definite Matrices

### Preserving low rank positive semidefinite matrices

Dominique Guillot, Apoorva Khare, Bala Rajaratnam

We consider the problem of characterizing real-valued functions $f$ which, when applied entrywise to matrices of a given rank, preserve positive semidefiniteness. Understanding if and how positivity is preserved in various settings has been the focus of a concerted effort throughout the past century. The problem has been studied most notably by Schoenberg and Rudin. One of their most significant results states that $f$ preserves the set of all positive semidefinite matrices, if and only if $f$ is absolutely monotonic on the positive axis, i.e., has a Taylor series with nonnegative coefficients.

In this work, we focus on functions preserving positivity for matrices of low rank. We obtain several new characterizations of these functions. Additionally, our techniques apply to classical problems such as the one mentioned above. In contrast to previous work, our approach is elementary, and enables us to provide intuitive, elegant, and enlightening proofs.

### Sparse positive definite matrices, graphs, and absolutely monotonic functions

Dominique Guillot, Apoorva Khare, Bala Rajaratnam

We study the problem of characterizing functions, which when applied entrywise, preserve the set of positive semidefinite matrices with zeroes according to a family $\mathcal{G} = \{G_n\}$ of graphs. The study of these and other maps preserving different notions of positivity has many modern manifestations and involves an interplay between linear algebra, graph theory, convexity, spectral theory, and harmonic analysis. In addition to the inherent theoretical interest, this problem has important consequences in machine learning (via kernels), in regularization of covariance matrices, and in the study of Toeplitz operators.

We obtain a characterization for $\mathcal{G}$ an arbitrary collection of trees; the only previous result along these lines was for all complete graphs: $\mathcal{G} = \{K_n\}$, in terms of absolutely monotonic functions. We discuss the case of fractional Hadamard powers and of polynomials with negative coefficients for trees. Finally, we find a stronger condition for preserving positivity for any sequence $\mathcal{G} = \{G_n\}$ of graphs with unbounded maximal degree, which is only satisfied by absolutely monotonic functions.

### Regularization of positive definite matrices: Connections between linear algebra, graph theory, and statistics

Bala Rajaratnam, Dominique Guillot, Brett Naul, Al Hero

Positive definite matrices arise naturally in many areas within mathematics and also feature extensively in scientific applications. In modern high-dimensional applications, a common approach to finding sparse positive definite matrices is to threshold their small off-diagonal elements. This thresholding, sometimes referred to as hard-thresholding, sets small elements to zero. Thresholding has the attractive property that the resulting matrices are sparse, and are thus easier to interpret and work with. In many applications, it is often required, and thus implicitly assumed, that thresholded matrices retain positive definiteness. We formally investigate the (linear) algebraic properties of positive definite matrices which are thresholded. We also undertake a detailed study of soft-thresholding, another important technique used in practice. Some interesting and unexpected results will be presented. Finally, we obtain a full characterization of general maps which preserve positivity when applied to off-diagonal elements, thereby extending previous work by Schoenberg and Rudin.

## Speaker Index

(minisymposium and contributed talks)